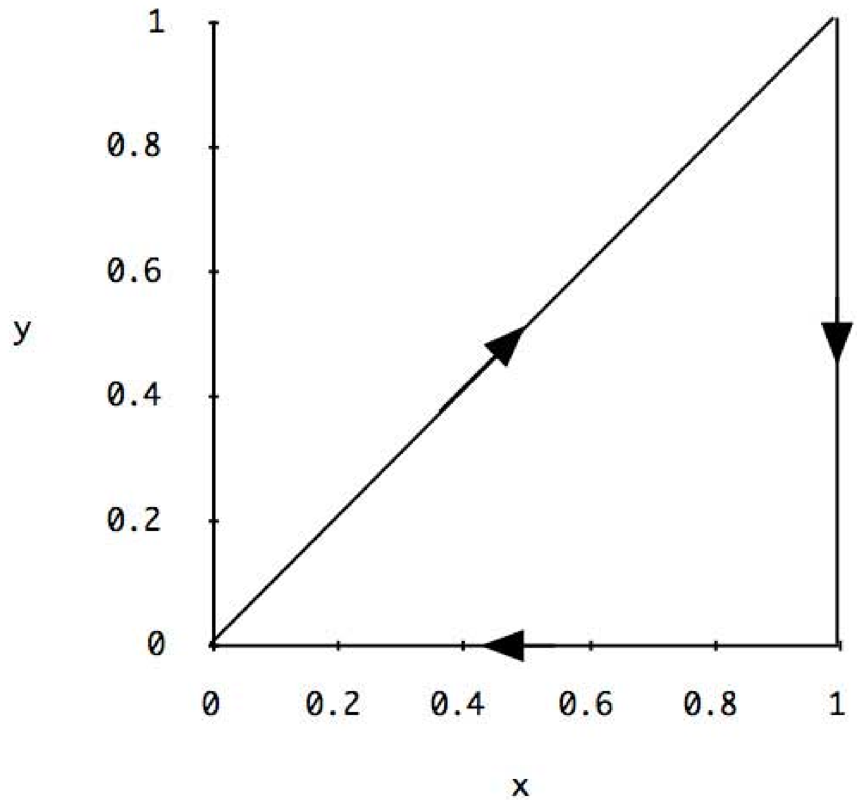
[13.4] - Green's theorem problems

1. Let C be the triangle path $(0,0) \to (1,1) \to (0,1) \to (0,0)$.

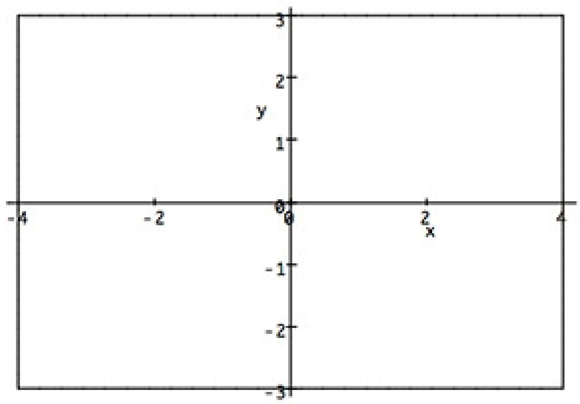


Then $\int_C 2y\,dx - 3x\,dy$ equals ??

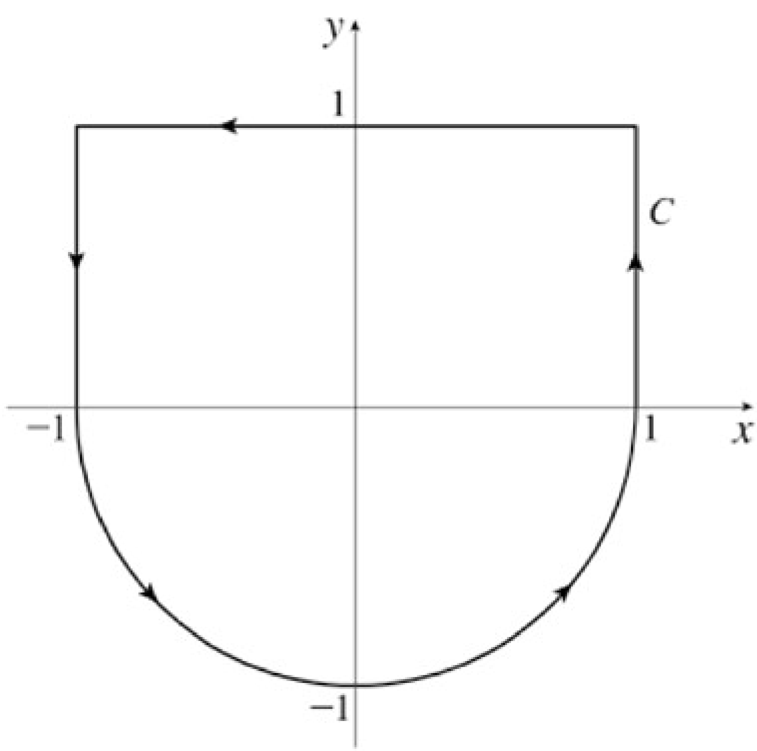
The orientation of the path is negative. The vector integral around the outside is equal to the positive orientation:

$$
\begin{aligned}
\oint_{-C} (-2y\,dx + 3x\,dy) = \\
\oint P\,dx + Q\,dy &= \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA \\
&= \iint (3 - (-2)) = 5 \iint dA = 5 * \frac{1}{2}.
\end{aligned} \tag{1}
$$

2. Use Green's Theorem to calculate $\oint_C (y - x)\,dx + (2x - y)\,dy$ where C is the boundary of the rectangle shown.

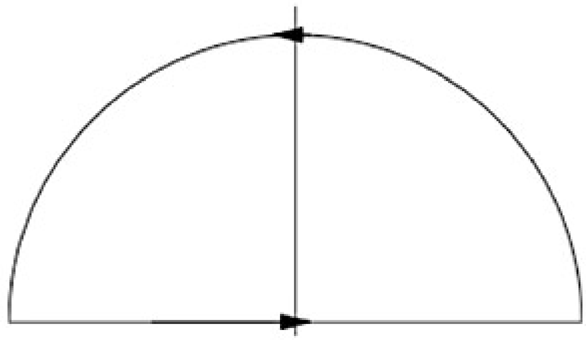


3. Compute $\oint_C \left(-\frac{xy^4}{2}\right)\,dx + \left(x^2y^3\right)\,dy$ where C is the curve shown below.

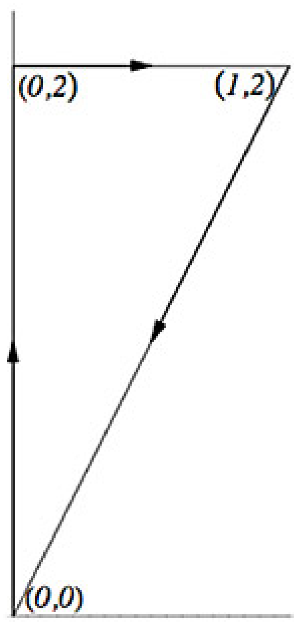


4. Use Green's Theorem to evaluate the following line integrals:

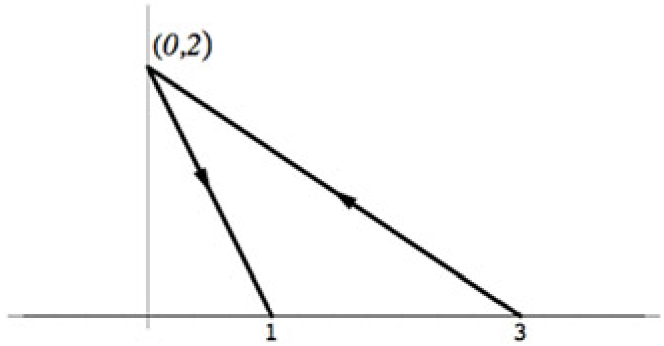
a. $\oint_C \left(\arctan(x^2) - y^2\right)\, dx + \left(x^2 y - \ln(y^2+1)\right)\, dy$ where C is the semicircle $y = \sqrt{4-x^2}$ together with the line segment $(-2,0) \to (2,0)$ as shown.



b. $\oint_C xy\, dx + (x^2+y^2)\, dy$ where C is this triangle.



5. Consider the ***non-closed*** curve C, $(3,0) \to (0,2) \to (1,0)$ as shown. Figure out a way to use Green's Theorem to help you compute $\int_C (x+y)\,dx + (3x-y)\,dy$.



Hint: