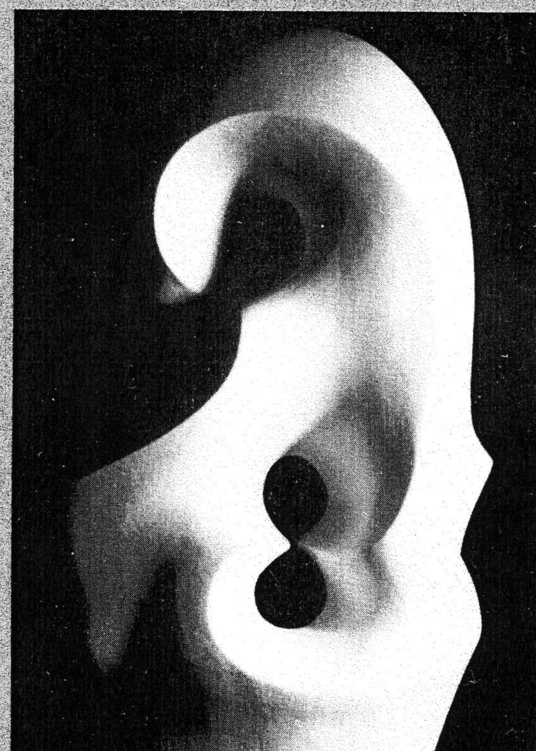
Mathematical Association of America

Models for Undergraduate Research in Mathematics



Lester Senechal, Editor

MAA Notes
Number 18

A Program Alumnus as Program Director

"What factors create a successful research experience for undergraduates? A good mathematical problem, a spirit of collaboration, and supportive interaction between students and faculty."

16 Observations From Both Sides of the Fence

David Housman
Worcester Polytechnic Institute [1]

I was a student participant in an undergraduate research program during the summer of 1977, under the direction of Joeseph Gallian. For the past two summers I, along with three colleagues, have directed an undergraduate research program. I have made no formal study of pedagogy, and after only five years of teaching experience, I consider myself a novice teacher. Yet, I believe that I have a few insights to offer from my experience on "both sides of the fence."

Before presenting my main observations, I will give a short summary of the basic structure of both summer programs, the one which I have directed, and the one in which I was a student participant. Before the summer programs began, students were selected on the basis of applications and assigned to a faculty advisor. The faculty advisor sent each of his or her students some reading material that included brief descriptions of potential problems to be considered during the summer. Students chose a problem from a selected list of fairly significant unsolved problems on which others were not too likely to be working. Students met individually with their faculty advisor at least once each week but usually more often. Students presented their progress in a weekly seminar to the other students and faculty. On other occasions, faculty advisors and other mathematicians presented talks for the students. Several social events involving both students and faculty were held. Final written reports are expected from each student. In both programs, it is hoped that papers will be submitted for publication sometime after the end of the summer, and that students will present their results at a professional meeting. The major differences between the two programs are the amount of academic and social interaction among the students and the amount of direct faculty advisor involvement in student research—our program has a greater amount in both areas. I think that the expectations were slightly higher in Gallian's program. Gallian's program had three faculty advisors and six students chosen from a nationwide applicant pool; the WPI program has four faculty advisors and nine students chosen from a New England and New York state applicant pool. Gallian's program lasted ten weeks, and the WPI program is in the fifth of eight weeks.

What factors create a successful research experience for undergraduates? That is, what nurtures a budding excitement for mathematics? First, the student needs to work on a good mathematical problem. Second, a spirit of collaboration needs to be developed. Finally, there must be extensive and supportive interaction between students and faculty. I will describe each of these in turn using anecdotal evidence.

First, the student needs to work on a good mathematical problem. By "good," I mean that the problem is interesting, accessible and flexible. I do not want to start a discussion of mathematical aesthetics, because the major reason a student finds a problem interesting is that her or his faculty advisor and others with whom she or he comes in contact find the problem interesting. Certainly, the problem must be non-trivial and new—at least to the student and faculty advisor. The student should have some role in the selection of the

[1]Present Address: Department of Mathematics and Computer Science, Drew University, Madison, NJ 09740.

problem. My experience is that the problem should sound "applied." How many flipping and rotating machines are needed to make sure that any envelope can be positioned with its stamp in a particular orientation? What is a fair way to allocate the costs of a joint venture? How can a structure made of rigid beams be constructed so that the structure is rigid even when a single beam is removed for repair? Can a fleet of airplanes be serviced in the available hangars? The student rapidly moves beyond the application to the abstraction. For example, the envelope problem for me became counting the number of Hamiltonian cycles in the Cayley diagram of certain classes of groups, and the rigidity problem has become finding circuits in the rigidity matroid of excess more than three. The importance of the supposed application is that it provides an initial image and it is a device for the student to explain what she or he is doing to her nonmathematician parents and friends. When this is lacking, the following situation can arise: one student expressed reservations about continuing with his work because of its "lack of applicability," even though he chose the problem based on his own literature review.

For a problem to be accessible, the statement and some ramifications must be understood rapidly, and a "straight-forward" approach should lead to substantial progress. This usually means that the statement of the problem uses very simple mathematical concepts and suggests an examination of examples. The importance of accessibility can be seen in what happens in its absence. The three students working on rigidity needed to understand a fair amount about matroid theory to understand the questions and make some progress towards solutions. The method used by the faculty advisor to impart the necessary knowledge involved reading, completion of a number of problems, and presentation of the results. With this approach, the research skills of discovery and presentation could be developed along the way. Still a certain level of frustration developed which was expressed by one student in the words, "Aren't we suppose to be trying to discover new things rather than rediscovering what is already known?" Flexibility on the part of the faculty advisor turned this crisis into a positive learning experience: Ari and Tricia became interested in some questions dealing with self-dual planar graphs and maximal planar graphs that were suggested by the preparatory material; Donnie spent a week on a game theory problem before returning, refreshed, to matroid theory. From day one (because of course work that occurred before the summer), Lori understood the problem she selected: are there allocation methods that are coalitionally rational and monotonic on three and four person games? In particular, does the nucleolus satisfy this property? The "straight-forward" approach soon developed into a set of over forty tedious cases in which no insight was being gained. The frustration here was not that the question was hard to understand but that all the easy approaches yielded no substantive progress.

By a flexible problem, I mean one that suggests several approaches to its solution and easily mutates into other problems: more tractible ones if no progress is being made or more general ones if the original question is answered quickly. My envelope question was one of optimization, but this naturally led to questions of identification and enumeration. Many problems are a single question that can be asked for different subclasses (e.g., groups and graphs). So, when Donna found calculating the nucleolus of minimal cost spanning tree games to require a background in linear programming which she did not possess, she was able to transfer much of her knowledge to calculating the nucleolus for 2-additive games and obtain some very interesting results, and now she hopes to extend these results (or at least the methods) to 3-additive games. The inaccessibility of Lori's problem could be seen as an inflexibility because of the initial lack of different approaches to handling the problem; even the related questions appeared to require the bankrupt approach already in use. What finally caused a breakthrough was the close examination of the cases already considered and the discovery of a mistake in a textbook result that had been important in earlier work.

A good problem is certainly not a sufficient requirement for a positive research experience. It is necessary that a spirit of collaboration be developed. By this I mean that the student must believe that what s/he is doing is valuable, that s/he can perform competently, and that s/he can give and accept suggestions and critique in a constructive manner. At the beginning of the summer, the faculty-student relationship is the usual unequal one where (from the student's perspective) the faculty advisor is an all-knowing semi-god who will impart, hopefully, a modicum of knowledge and wisdom to this slightly frightened person who undervalues her or his own experience and capabilities. The goal is for the student to see herself or himself as an equal. Maybe the faculty advisor does have more experience and training, but the different background, fresh approach and energy of the student can make her or his contribution original and significant. By the end of the summer, the student should believe that s/he could fill the advisor's shoes one day (I believed this at the end of my summer with Joe).

This sounds a lot like maturation to adulthood. Indeed, I would argue that it is a second level of mathematical maturity, the first level being the ability to understand and recreate mathematical proofs (the major goal of most undergraduate programs in mathematics). Just as the path to adulthood requires a passage through the teenage years, the path to collaboration requires that the student assert her independence. There is a point when the student is convinced that *she* did it. This was certainly the case when I discovered a graphical representation of group generating sequences, when Tricia discovered a formula for the number of triangles in maximal planar graphs, when Donna discovered the short and elegant argument that gave the Shapley value of 2-additive games, when Diane started finding counterexamples to all of my conjectures, and when Lori made an insightful conjecture about the conditions under which nucleolus nonmonotonicity would occur. Even learning later that their "discovery" is not new or even correct does not seem to dampen the effect of this initial self-discovery. For example, I learned near the end of the summer that my "original" idea for representing group generating sequences in a graphical manner was the long known concept of a Cayley diagram. Tricia found that her equality triangle formula was really an inequality and had been discovered in 1979. Diane subsequently found a counterexample to one of her own conjectures. In all three situations, there was a certain amount of disappointment, but the overall reaction was to use the new knowledge to motivate new questions and further work rather than to give up.

In my experience, the need to assert one's independent worth has extended beyond the academic sphere. In both programs, a number of social events were planned by the faculty. Nonetheless, the students reciprocated in both programs with a dinner party for everyone.

We found a somewhat unusual confidence builder. A faculty enhancement workshop in discrete mathematics was offered near where our program was being held, and we were able to have our students attend talks given by Fred Roberts and Newton Garber. The glow on the students' faces after Robert's talk said, "Wow! I understood these talks just as well as the faculty present!" One has to be careful though. My three students and I attended