

INFINITE PLAYER NONCOOPERATIVE GAMES AND THE CONTINUITY OF THE NASH EQUILIBRIUM CORRESPONDENCE*

DAVID HOUSMAN

Worcester Polytechnic Institute

The usual definition of a noncooperative game is extended in two different ways: first, by replacing the finite player set with a measure space, and second, by eliminating the player set and considering a distribution of the players' characteristics. Feasible strategy profiles and Nash equilibria obtained from the two approaches are compared. The feasible strategy profile correspondence is shown to be continuous. The Nash equilibrium correspondence is shown to be upper hemicontinuous and nearly lower hemicontinuous on the class of convex and equicontinuous games. These results show when it is reasonable to use an infinite player game as an approximation of a large, but finite, player game.

1. Introduction. At times it is desirable to understand the strategic behavior of a large number of interacting agents. For example, the economy or political mechanisms of an entire country, commuter traffic patterns in a large city, or shareholder control of a large corporation. In such situations it is natural to posit a continuum of agents so that the mathematics is more tractable. Aumann's study of markets [1] is a classic example using cooperative games; Kannai and Peleg [11], Nti [14], and Housman [8] are examples using noncooperative games. An assumption inherent in such an approach is that the continuous player model accurately represents the large, but finite, player reality. Kannai [10] studied the validity of this assumption in the context of market games by considering the continuity properties of the core correspondence on the space of market games. This paper is in the spirit of [10], but is concerned with the Nash equilibrium correspondence on the space of noncooperative games.

We first generalize the definition of a finite player noncooperative game in two different ways so that games with an infinite number of players can be considered. In §3, this is accomplished by replacing the finite set of indices with a measure space (this is the approach taken by Schmeidler [15]). In §4, this is accomplished by considering the distribution of player characteristics (this is the approach taken by Mas-Colell [13]). §4 also develops the relationship between these two generalizations in a manner similar to the work of Hart, Hildenbrand and Kohlberg [7] on market economies. The models formulated here generalize those of Schmeidler [15] and Mas-Colell [13] in one significant respect: individual players can have a positive influence on other players' payoffs. Mathematically, we allow the measure space to contain singletons and one characteristic that a player possesses is her measure as a singleton. These more general models are useful when a few of the millions of stockholders of a corporation hold a significant proportion of the stock, or in an economy of oligopolist producers and many consumers.

*Received March 24, 1986; revised March 30, 1987.

AMS 1980 subject classification. Primary: 90D13.

IAOR 1973 subject classification. Main: Games.

OR/MS Index 1978 subject classification. Primary: 238 Games/noncooperative.

Key words. Infinite player games, Nash equilibrium, continuity of game solutions.

