

Enumeration of hamiltonian paths in Cayley diagrams

DAVID HOUSMAN

Abstract. Let G be a group generated by a subset of elements S . The Cayley diagram of G given S is the labeled directed graph with vertices identified with the elements of G and (v, u) is an edge labeled h if $h \in S$ and $uh = v$. The sequence of elements of S corresponding to the edges transversed in a hamiltonian path (whose initial vertex is the identity) is called a group generating sequence (abbreviated ggs) in S .

In this paper a minimal upper bound for the number of ggs's in a pair of generator elements for any two-generated group is given. For all groups of the form $G = \langle a, b : b^n = 1, a^m = b^r, ba = ab^{-1} \rangle$ where m is even, it is shown that the number of ggs's in $\{a, b\}$ is $1 + m(n-1)/2$. An algorithm is developed that yields the number of ggs's for two-generated groups $G = \langle a, b \rangle$ for which $\langle ba^{-1} \rangle \triangleleft G$. Explicit forms for the counted ggs's are also provided.

1. Introduction

Let G be a group generated by a subset of elements S . The Cayley diagram of G given S , denoted $D_S(G)$, is the labeled directed graph with vertices identified with the elements of G and (v, u) an edge labeled h if $h \in S$ and $uh = v$.

The existence of hamiltonian paths and circuits in Cayley diagrams has been studied by several authors. Rankin [11, 12] has provided some sufficient and necessary conditions on the group and generating set. Holsztyński and Nathanson [see 4] showed that there exists a hamiltonian path in every Cayley diagram of an abelian or hamiltonian group or a group whose order is no more than 15. Witte [14] proved the same for groups having a cyclic subgroup of index 2. However, there are Cayley diagrams having no hamiltonian path. Nijenhuis and Wilf [10, pp. 288-289] gave the example of the symmetric group on five letters given the generating set $\{(1\ 2), (1\ 2\ 3\ 4\ 5)\}$. According to Nathanson [9], Milnor constructed a solvable group having no hamiltonian path.

Necessary and sufficient conditions for a hamiltonian circuit to exist in a

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