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AMERICAN MATHEMATICAL SOCIETY
Cooperative Game Theory REU

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The Drew University REU in Cooperative Game Theory was held during the summers of 1990-1993. The program brought together groups of six students (three in the final summer) for a first-time immersion in mathematical research. The program was designed to create a community of scholars where inquiry was valued and questions of career and life aspirations could be discussed in an atmosphere of care and trust. The major components of the program are described below.

Students. I sought mathematically capable students for whom the program would have a major impact on their career decisions. Operationally, this meant students who had (1) completed at least two courses in which they had written proofs, (2) strong letters of recommendation, (3) expressed openness to, but uncertainty of, graduate studies, and (4) no substantial research experience. Since more applicants fit these criteria than the available positions, final selections were made to ensure a diversity of mathematical, collegiate, and social backgrounds. About equal numbers of students came from baccalaureate colleges, masters institutions, and doctoral universities.

Faculty. I was the program director and sole mentor. I met with each student individually from one to five hours weekly. This usually meant a discussion in my office, but we also had discussions with each other over meals, in their house, between talks and in the dorms at conferences, in vehicles on our way to and from conferences, and on exploratory walks. Students also interacted with faculty visitors and with faculty at conferences we attended as a group. Students found close formal and informal interaction with faculty and students to be the most beneficial aspect of the program. Through these interactions the major gains were made in each student's level of mathematical

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maturity, ability to express mathematics, amount of self-confidence, and desire for a mathematical career.

**Topics.** The general area of research was axiomatic and algorithmic aspects of values for cooperative games and their applications. Students were introduced to (1) games in coalitional function form and how this model could be modified in various ways, (2) several standard solution concepts, (3) several properties games and solutions could possess, (4) some known interrelationships, and (5) some interesting unanswered questions. With consultation, students were then free to choose a topic. By selecting different topics, each student felt ownership and developed expertise. With one general area of research, the students were able to understand and support the progress of each other. By providing a framework for the research area, students were empowered to ask their own questions as well as making conjectures and proving results.

**Seminars.** I started out each summer with four lectures and group discussions that introduced cooperative game theory and the area of research. There were weekly seminars where each student talked about her or his progress and answered questions for 10 to 30 minutes. Many seminars included a visitor who listened and reacted to student talks, gave a talk of his or her own, and discussed career and graduate school over a meal. The small and supportive audience provided a friendly environment for students to gain confidence in and improve their speaking skills. At the same time, valuable suggestions were raised and students were able to clarify the direction of their research.

**Professional Trips.** During each summer, we traveled as a group to Rutgers University, to an international conference on game theory, and the summer MAA/PME meetings (expect in 1990). The meetings provided opportunities formal student presentations and for interaction with faculty having common interests.

**Reports.** Students wrote both an interim and a final report. The interim report forced students to clearly describe their research topics and allowed me to assist student development of proper style early in the summer. The final reports described the problem considered, background literature read, approach(es) taken, results obtained, and questions motivated by the results.

**Residence.** During the first three summers, students shared an on-campus house with separate bedrooms, adequate kitchen facilities, and a large living room and outside deck. The house provided pleasant space for both private study and group interaction.

**Social Events.** I favor informal events where conversation flows easily, and so there were several luncheons and dinners in restaurants
and my home. We also went on a one day excursion to Manhattan each summer.

**Continuation.** My last official communication with each student included written comments about their final reports and a draft recommendation letter. I have found that the second item is particularly valuable for the participants because it gives them written confirmation of their strengths, while the first item tends to point out areas needing improvement. I have remained in somewhat irregular contact with these 21 students as well as students whom I have mentored at other institutions. I know that five have received doctoral degrees and five are in their dissertation stage of doctoral studies in a variety of mathematical sciences programs.

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