A Biological Auction
Valparaiso University Talk

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July 8, 2010
A strange auction
A strange auction

The biological connection
Outline

- A strange auction
- The biological connection
- A strange auction repeated
Outlook

- A strange auction
- The biological connection
- A strange auction repeated
- Best response to a known opponent
Outline

- A strange auction
- The biological connection
- A strange auction repeated
- Best response to a known opponent
- Biologically optimal strategy
Outline

- A strange auction
- The biological connection
- A strange auction repeated
- Best response to a known opponent
- Biologically optimal strategy
- Concluding remarks
A Strange Auction

- Open ascending bid auction for a prize.
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- The highest bidder wins the prize but pays her bid.
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- The highest bidder wins the prize but pays her bid.
- The second highest bidder wins nothing but pays his bid.
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- No one else pays.
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- Biological interpretation.
The value of the prize to you is on the paper and was drawn from a uniform distribution on 0 to 1000.
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Sealed (nonnegative) bid auction for the prize.
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Sealed (nonnegative) bid auction for the prize.

Both of us pay the lower bid, but only the higher bidder wins the prize.
A Strange Auction Repeated

- The value of the prize to you is on the paper and was drawn from a uniform distribution on 0 to 1000.
- Sealed (nonnegative) bid auction for the prize.
- Both of us pay the lower bid, but only the higher bidder wins the prize.
- Repeat 30 times with a variety of opponents.
A Strange Auction Repeated

- The value of the prize to you is on the paper and was drawn from a uniform distribution on 0 to 1000.
- Sealed (nonnegative) bid auction for the prize.
- Both of us pay the lower bid, but only the higher bidder wins the prize.
- Repeat 30 times with a variety of opponents.
- Keep track of the strategy you use and its effectiveness.
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- Both of us pay the lower bid, but only the higher bidder wins the prize.
- Repeat 30 times with a variety of opponents.
- Keep track of the strategy you use and its effectiveness.
- Play now!

- What were the most effective strategies?
Strange Auction Model I

- Both players pay the lower bid, but only the higher bidder wins the prize.
Strange Auction Model I

- Both players pay the lower bid, but only the higher bidder wins the prize.
- A player knows what the prize is worth to him/her but not what it is worth to his/her opponent.

$f(v)$ is the probability density the prize is worth $v$ to a player.
Strange Auction Model I

- Both players pay the lower bid, but only the higher bidder wins the prize.
- A player knows what the prize is worth to him/her but not what it is worth to his/her opponent.
- $f(v)$ is the probability density the prize is worth $v$ to a player.

![Graph showing $f(v) = 1$ and $F(v) = v$.]
Strange Auction Model I

- Both players pay the lower bid, but only the higher bidder wins the prize.
- A player knows what the prize is worth to him/her but not what it is worth to his/her opponent.
- \( f(v) \) is the probability density the prize is worth \( v \) to a player.
- \( \beta(v) \) is the opponent’s bid if the prize is worth \( v \) to him.

![Graph showing bid vs. value](image-url)
Strange Auction Model I

- Both players pay the lower bid, but only the higher bidder wins the prize.
- A player knows what the prize is worth to him/her but not what it is worth to his/her opponent.
- $f(v)$ is the probability density the prize is worth $v$ to a player.
- $\beta(v)$ is the opponent’s bid if the prize is worth $v$ to him.

If I value the prize at $v$ and bid $b$, my expected payoff is

$$\pi(b) = \int_{\beta(u)<b} (v - \beta(u))f(u) \, du - b \int_{\beta(u)\geq b} f(u) \, du$$
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I want to choose $b \geq 0$ to maximize $\pi(b)$. 
Maximize the following at $b = b^*$:

$$\pi(b) = \int_{\beta(u) < b} (v - \beta(u)) f(u) \, du - b \int_{\beta(u) \geq b} f(u) \, du$$
Payoff Maximization (General Case)

- Maximize the following at $b = b^*$:

$$\pi(b) = \int_{\beta(u)<b} (v - \beta(u)) f(u) \, du - b \int_{\beta(u)\geq b} f(u) \, du$$

- Assume $\beta$ is strictly increasing and $F$ is the cdf of $f$.

$$\pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u)) f(u) \, du - b(1 - F(\beta^{-1}(b)))$$
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  $$\pi(b) = \int_{0}^{\beta^{-1}(b)} (v - \beta(u))f(u) \, du - b(1 - F(\beta^{-1}(b)))$$

- Assume $\beta$ is differentiable.

  $$\pi'(b) = \frac{(v - \beta(\beta^{-1}(b)))f(\beta^{-1}(b))}{\beta'(\beta^{-1}(b)))} - (1 - F(\beta^{-1}(b))) + \frac{bf(\beta^{-1}(b))}{\beta'(\beta^{-1}(b)))}$$
Payoff Maximization (General Case)

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  \]

- Simplify.
  \[
  \pi'(b) = vf(\beta^{-1}(b))/\beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))
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Payoff Maximization (General Case)

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- Simplify.
  \[
  \pi'(b) = vf(\beta^{-1}(b))/\beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))
  \]

- First order necessary condition $\pi'(b^*) = 0$.
  \[
  0 = vf(\beta^{-1}(b^*))/\beta'(\beta^{-1}(b^*)) - (1 - F(\beta^{-1}(b^*)))
  \]
Payoff Maximization (Special Case)

- First order necessary condition.

\[ 0 = \pi'(b^*) = vf(\beta^{-1}(b^*)) / \beta'(\beta^{-1}(b^*))) - (1 - F(\beta^{-1}(b^*))) \]
Payoff Maximization (Special Case)

- First order necessary condition.

\[ 0 = \pi'(b^*) = vf(\beta^{-1}(b^*)) / \beta'(\beta^{-1}(b^*))) - (1 - F(\beta^{-1}(b^*))) \]

- Suppose \( f(v) = 1, v \in [0, 1] \) and \( \beta(v) = av, v \in [0, 1] \). Hence, \( F(v) = v, v \in [0, 1] \) and \( \beta^{-1}(b) = b/a, b \in [0, a] \).

\[ 0 = v \cdot 1/a - (1 - b^*/a) \]
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- Solve for \( b^* \).

  \[ b^* = a - v \]
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  \[ b^* = a - v \]

- We have found a local minimum!
  \[ \pi'(b) = v/a - 1 + b/a \]
  \[ \pi(b) = (v/a - 1)b + (1/2a)b^2 \]
Payoff Maximization (Special Case)

- First order necessary condition.
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  \[ \pi'(b) = v/a - 1 + b/a \]
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- The correct maximum is a trigger strategy.
  \[ b^* = \begin{cases} 
    0, & \text{if } v \leq a/2 \\
    a, & \text{if } v \geq a/2 
  \end{cases} \]
Strange Auction Model II

- Both players pay the lower bid, but only the higher bidder wins the prize.
- A player knows what the prize is worth to him/her but not what it is worth to his/her opponent.
- $f(v)$ is the probability density the prize is worth $v$ to a player.
- $\beta(v)$ is the opponent’s bid if the prize is worth $v$ to him.

If I value the prize at $v$ and bid $b$, my expected payoff is

$$\pi(b) = \int_{\beta(u)<b} (v - \beta(u)) f(u) \, du - b \int_{\beta(u)\geq b} f(u) \, du$$

- Assume $\beta(v)$ is the player’s payoff maximizing bid, that is,

$$\pi(\beta(v)) \geq \pi(b)$$

for all $b \geq 0$. 
Payoff Maximization (General Case)

Maximize the following at $b = \beta(v)$:

$$\pi(b) = \int_{\beta(u) < b} (v - \beta(u))f(u) \, du - b \int_{\beta(u) \geq b} f(u) \, du$$
Maximize the following at $b = \beta(v)$:

$$\pi(b) = \int_{\beta(u)<b} (v - \beta(u)) f(u) \, du - b \int_{\beta(u) \geq b} f(u) \, du$$

As before, take the derivative.

$$\pi'(b) = vf(\beta^{-1}(b)) / \beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))$$
Maximize the following at \( b = \beta(v) \):

\[
\pi(b) = \int_{\beta(u)<b} (v - \beta(u)) f(u) \, du - b \int_{\beta(u)\geq b} f(u) \, du
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As before, take the derivative.

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\pi'(b) = vf(\beta^{-1}(b)) / \beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))
\]

First order necessary condition \( \pi'(\beta(v)) = 0 \).

\[
0 = vf(v) / \beta'(v) - (1 - F(v))
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Payoff Maximization (General Case)

- Maximize the following at \( b = \beta(v) \):
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- Solve for \( \beta' \).
  \[
  \beta'(v) = \frac{vf(v)}{1 - F(v)}
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- Solve for \( \beta' \).

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\beta'(v) = \frac{vf(v)}{1 - F(v)}
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- Solve for \( \beta \).

\[
\beta(v) = \int_{0}^{v} \frac{uf(u)}{1 - F(u)} \, du
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Payoff Maximization (General Case)

- Maximize the following at $b = \beta(v)$:
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  \pi(b) = \int_{\beta(u)<b} (v - \beta(u))f(u)\,du - b\int_{\beta(u)\geq b} f(u)\,du
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- As before, take the derivative.
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  \pi'(b) = vf(\beta^{-1}(b)) / \beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))
  \]

- First order necessary condition $\pi'(\beta(v)) = 0$.
  \[
  0 = vf(v) / \beta'(v) - (1 - F(v))
  \]

- Solve for $\beta'$.
  \[
  \beta'(v) = \frac{vf(v)}{1 - F(v)}
  \]

- Solve for $\beta$.
  \[
  \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)}\,du
  \]

- This function is differentiable and increasing from $\beta(0) = 0$. 
Payoff Maximization (Special Case)

- Suppose $f(u) = 1, u \in [0, 1]$ and $F(u) = u, u \in [0, 1]$. 
Payoff Maximization (Special Case)

- Suppose \( f(u) = 1, \ u \in [0, 1] \) and \( F(u) = u, \ u \in [0, 1] \).
- \( \beta(v) = \int_0^v \frac{uf(u)}{1-F(u)} \, du = \int_0^v \frac{u}{1-u} \, du = -v - \ln(1 - v) \).
Suppose $f(u) = 1, \ u \in [0, 1]$ and $F(u) = u, \ u \in [0, 1]$.

$$\beta(v) = \int_0^v \frac{uf(u)}{1-F(u)} \, du = \int_0^v \frac{u}{1-u} \, du = -v - \ln(1-v).$$

$$\pi_{\text{max}}(v) = \frac{1}{2} v^2.$$
To verify we have found a maximum, substitute

$$\beta'(v) = \frac{vf(v)}{1 - F(v)}$$
To verify we have found a maximum, substitute

$$\beta'(v) = \frac{v f(v)}{1 - F(v)}$$

into

$$\pi'(b) = vf(\beta^{-1}(b))/\beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))$$
To verify we have found a maximum, substitute

\[ \beta'(v) = \frac{vf(v)}{1 - F(v)} \]

into

\[ \pi'(b) = vf(\beta^{-1}(b)) / \beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b))) \]

to obtain

\[ \pi'(b) = (1 - F(\beta^{-1}(b))(v / \beta^{-1}(b) - 1) \]
To verify we have found a maximum, substitute

$$\beta'(v) = \frac{vf(v)}{1 - F(v)}$$

into

$$\pi'(b) = vf(\beta^{-1}(b)) / \beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))$$

to obtain

$$\pi'(b) = (1 - F(\beta^{-1}(b))(v / \beta^{-1}(b) - 1)$$

which is positive if $b < \beta(v)$
To verify we have found a maximum, substitute

$$\beta'(v) = \frac{vf(v)}{1 - F(v)}$$

into

$$\pi'(b) = vf(\beta^{-1}(b))/\beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))$$

to obtain

$$\pi'(b) = (1 - F(\beta^{-1}(b)))(v/\beta^{-1}(b) - 1)$$

which is positive if $b < \beta(v)$

and negative if $b > \beta(v)$. 
Payoff Using the Strategy

- The payoff to a player who values the prize at $v$ and bids $b$

$$\pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u)) f(u) \, du - b(1 - F(\beta^{-1}(b)))$$
Payoff Using the Strategy

The payoff to a player who values the prize at $v$ and bids $b$

$$\pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u))f(u) \, du - b(1 - F(\beta^{-1}(b)))$$

is maximized at $b = \beta(v)$

$$\pi_{\text{max}}(v) = \int_0^{v} (v - \beta(u))f(u) \, du - \beta(v)(1 - F(v))$$
Payoff Using the Strategy

- The payoff to a player who values the prize at $v$ and bids $b$
  \[ \pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u))f(u)\,du - b(1 - F(\beta^{-1}(b))) \]

- is maximized at $b = \beta(v)$
  \[ \pi_{\text{max}}(v) = \int_0^v (v - \beta(u))f(u)\,du - \beta(v)(1 - F(v)) \]

- Hence,
  \[ \pi_{\text{max}}(0) = 0 \]
Payoff Using the Strategy

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$$\pi(b) = \int_{0}^{\beta^{-1}(b)} (v - \beta(u)) f(u) \, du - b(1 - F(\beta^{-1}(b)))$$

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$$\pi_{\text{max}}(v) = \int_{0}^{v} (v - \beta(u)) f(u) \, du - \beta(v)(1 - F(v))$$

- Hence,

$$\pi_{\text{max}}(0) = 0$$

- Taking the derivative

$$\pi'_{\text{max}}(v) = (v - \beta(v))f(v) + F(v) - \beta'(v)(1 - F(v)) + \beta(v)f(v)$$
Payoff Using the Strategy

- The payoff to a player who values the prize at \( v \) and bids \( b \)
  \[
  \pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u)) f(u) \, du - b(1 - F(\beta^{-1}(b)))
  \]

- is maximized at \( b = \beta(v) \)
  \[
  \pi_{\text{max}}(v) = \int_0^v (v - \beta(u)) f(u) \, du - \beta(v)(1 - F(v))
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- Taking the derivative

$$\pi'_{\text{max}}(v) = (v - \beta(v))f(v) + F(v) - \beta'(v)(1 - F(v)) + \beta(v)f(v)$$

$$= vf(v) + F(v) - \frac{vf(v)}{1 - F(v)}(1 - F(v))$$
Payoff Using the Strategy

- The payoff to a player who values the prize at $v$ and bids $b$

$$\pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u)) f(u) \, du - b(1 - F(\beta^{-1}(b)))$$

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- Taking the derivative

$$\pi'_{\text{max}}(v) = (v - \beta(v)) f(v) + F(v) - \beta'(v)(1 - F(v)) + \beta(v) f(v)$$

$$= vf(v) + F(v) - \frac{vf(v)}{1 - F(v)}(1 - F(v))$$

$$= F(v) \geq 0$$
Payoff Using the Strategy

- The payoff to a player who values the prize at $v$ and bids $b$

$$
\pi(b) = \int_0^{\beta^{-1}(b)} (v - \beta(u))f(u) \, du - b(1 - F(\beta^{-1}(b)))
$$

- is maximized at $b = \beta(v)$

$$
\pi_{\text{max}}(v) = \int_0^v (v - \beta(u))f(u) \, du - \beta(v)(1 - F(v))
$$

- Hence,

$$
\pi_{\text{max}}(0) = 0
$$

- Taking the derivative

$$
\pi'_{\text{max}}(v) = (v - \beta(v))f(v) + F(v) - \beta'(v)(1 - F(v)) + \beta(v)f(v)
$$

$$
= vf(v) + F(v) - \frac{vf(v)}{1 - F(v)}(1 - F(v))
$$

$$
= F(v) \geq 0
$$

- The more you value the prize, the higher your expected payoff.
Recall the optimal bidding strategy.

\[ \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)} \, du \]
Surprising Observation

- Recall the optimal bidding strategy.

\[ \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)} \, du \]

- Find the average bid.

\[ \int_0^\infty \beta(v)f(v) \, dv = \int_0^\infty \int_0^v \frac{uf(u)}{1 - F(u)} \, du \, f(v) \, dv \]
Surprising Observation

- Recall the optimal bidding strategy.
  \[ \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)} \, du \]

- Find the average bid.
  \[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \int_0^v \frac{uf(u)}{1 - F(u)} \, du \, f(v) \, dv \]

- Interchange integrals \((0 \leq u \leq v < \infty)\).
  \[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \frac{uf(u)}{1 - F(u)} \int_u^\infty f(v) \, dv \, du \]
Surprising Observation

- Recall the optimal bidding strategy.

\[ \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)} \, du \]

- Find the average bid.

\[ \int_0^\infty \beta(v)f(v) \, dv = \int_0^\infty \int_0^v \frac{uf(u)}{1 - F(u)} \, du \, f(v) \, dv \]

- Interchange integrals (\(0 \leq u \leq v < \infty\)).

\[ \int_0^\infty \beta(v)f(v) \, dv = \int_0^\infty \frac{uf(u)}{1 - F(u)} \int_u^\infty f(v) \, dv \, du \]

- Since the inner integral is \(1 - F(u)\),

\[ \int_0^\infty \beta(v)f(v) \, dv = \int_0^\infty uf(u) \, du \]
Recall the optimal bidding strategy.

\[ \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)} \, du \]

Find the average bid.

\[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \int_0^v \frac{uf(u)}{1 - F(u)} \, du \, f(v) \, dv \]

Interchange integrals (\(0 \leq u \leq v < \infty\)).

\[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \int_u^\infty f(v) \, dv \, du \]

Since the inner integral is \(1 - F(u)\),

\[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty uf(u) \, du \]

The average bid equals the average value.
Surprising Observation

- Recall the optimal bidding strategy.
  \[ \beta(v) = \int_0^v \frac{uf(u)}{1 - F(u)} \, du \]

- Find the average bid.
  \[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \int_0^v \frac{uf(u)}{1 - F(u)} \, du \, f(v) \, dv \]

- Interchange integrals (0 ≤ u ≤ v < ∞).
  \[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \frac{uf(u)}{1 - F(u)} \int_u^\infty f(v) \, dv \, du \]

- Since the inner integral is 1 − F(u),
  \[ \int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty uf(u) \, du \]

- The average bid equals the average value.
- For some prize values v, the bid \( \beta(v) \) is greater than the value!
Concluding Remarks

Find the probability that the prize is won and too much is paid.

Repeat the analysis if only the winner pays the lower bid.

Repeat the analysis if only the winner pays the higher bid.
Find the probability that the prize is won and too much is paid.
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Repeat the analysis if only the winner pays the lower bid.
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Repeat the analysis if only the winner pays the lower bid.

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Questions?

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