A Biological Auction

Valparaiso University Talk

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Outline

- A strange auction

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- A strange auction
- The biological connection

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- A strange auction
- The biological connection
- A strange auction repeated

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- Best response to a known opponent

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- Biologically optimal strategy

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- Concluding remarks

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- Keep track of the strategy you use and its effectiveness.
- Play now!
- What were the most effective strategies?

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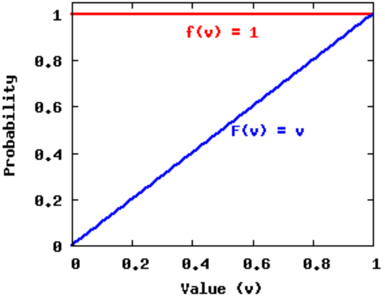
- $f(v)$ is the probability density the prize is worth v to a player.



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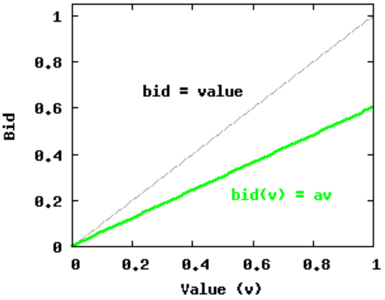

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- $\beta(v)$ is the opponent's bid if the prize is worth v to him.
- If I value the prize at v and bid b, my expected payoff is

$$\pi(b) = \int_{\beta(u)<b} (v - \beta(u)) f(u)\, du - b \int_{\beta(u)\geq b} f(u)\, du$$

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- I want to choose $b \geq 0$ to maximize $\pi(b)$.

Payoff Maximization (General Case)

- Maximize the following at $b = b^*$:

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- Simplify.

$$\pi'(b) = vf(\beta^{-1}(b))/\beta'(\beta^{-1}(b))) - (1 - F(\beta^{-1}(b)))$$

- First order necessary condition $\pi'(b^*) = 0$.

$$0 = vf(\beta^{-1}(b^*))/\beta'(\beta^{-1}(b^*))) - (1 - F(\beta^{-1}(b^*)))$$

Payoff Maximization (Special Case)

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- Suppose $f(v) = 1, v \in [0,1]$ and $\beta(v) = av, v \in [0,1]$. Hence, $F(v) = v, v \in [0,1]$ and $\beta^{-1}(b) = b/a, b \in [0,a]$.

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- We have found a local minimum!

$$\begin{aligned} \pi'(b) &= v/a - 1 + b/a \\ \pi(b) &= (v/a - 1)b + (1/2a)b^2 \end{aligned}$$

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- The correct maximum is a trigger strategy.

$$b^* = \begin{cases} 0, & \text{if } v \leq a/2 \\ a, & \text{if } v \geq a/2 \end{cases}$$

Strange Auction Model II

- Both players pay the lower bid, but only the higher bidder wins the prize.
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$$\pi(b) = \int_{\beta(u)<b} (v - \beta(u)) f(u)\, du - b \int_{\beta(u)\geq b} f(u)\, du$$

- Assume $\beta(v)$ is the player's payoff maximizing bid, that is,

$$\pi(\beta(v)) \geq \pi(b)$$

for all $b \geq 0$.

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- This function is differentiable and increasing from $\beta(0) = 0$.

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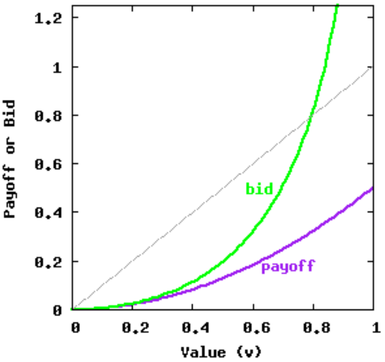
- $\pi_{\max}(v) = \frac{1}{2}v^2$.



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- which is positive if $b < \beta(v)$
- and negative if $b > \beta(v)$.

Payoff Using the Strategy

- The payoff to a player who values the prize at v and bids b

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- Taking the derivative

$$\pi'_{\max}(v) = (v - \beta(v)) f(v) + F(v) - \beta'(v)(1 - F(v)) + \beta(v) f(v)$$

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&= F(v) \geq 0
\end{aligned}$$

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- The more you value the prize, the higher your expected payoff.

Surprising Observation

- Recall the optimal bidding strategy.

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- Interchange integrals $(0 \leq u \leq v < \infty)$.

$$\int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty \frac{uf(u)}{1 - F(u)} \int_u^\infty f(v) \, dv \, du$$

- Since the inner integral is $1 - F(u)$,

$$\int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty uf(u) \, du$$

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- Interchange integrals ($0 \leq u \leq v < \infty$).

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- Since the inner integral is $1 - F(u)$,

$$\int_0^\infty \beta(v) f(v) \, dv = \int_0^\infty uf(u) \, du$$

- The average bid equals the average value.

Surprising Observation

- Recall the optimal bidding strategy.

$$\beta(v) = \int_0^v \frac{uf(u)}{1-F(u)}\, du$$

- Find the average bid.

$$\int_0^\infty \beta(v) f(v)\, dv = \int_0^\infty \int_0^v \frac{uf(u)}{1-F(u)} du\ f(v)\, dv$$

- Interchange integrals ($0 \leq u \leq v < \infty$).

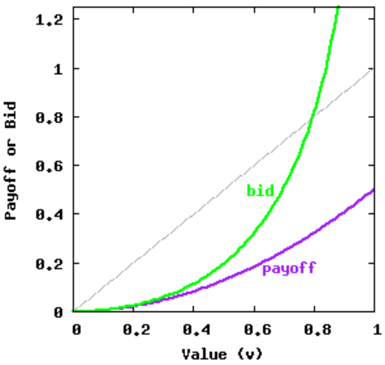
$$\int_0^\infty \beta(v) f(v)\, dv = \int_0^\infty \frac{uf(u)}{1-F(u)} \int_u^\infty f(v)\, dv\, du$$

- Since the inner integral is $1 - F(u)$,

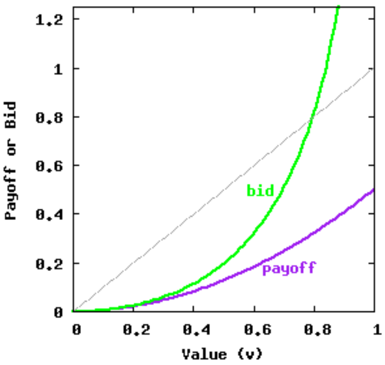
$$\int_0^\infty \beta(v) f(v)\, dv = \int_0^\infty uf(u)\, du$$

- The average bid equals the average value.
- For some prize values v, the bid $\beta(v)$ is greater than the value!

Concluding Remarks

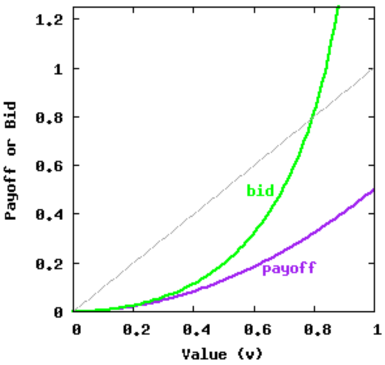


Concluding Remarks



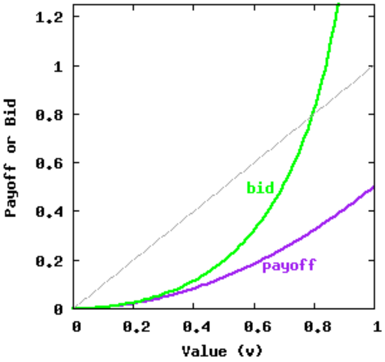
- Find the probability that the prize is won and too much is paid.

Concluding Remarks



- Find the probability that the prize is won and too much is paid.
- Repeat the analysis if only the winner pays the lower bid.

Concluding Remarks



- Find the probability that the prize is won and too much is paid.
- Repeat the analysis if only the winner pays the lower bid.
- Repeat the analysis if only the winner pays the higher bid.

Questions?

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