Games Mathematicians Play

Mathematical Models
of

Strategic Interaction

The Players

►Some are playing for donations to their favorite nonprofit organization:

- Ryan Moyer for Nothing but Nets
- Aaron Kaufmann for The Water Project
- Ben Sutter for The Correspondent
- Bruck Mulat for the American Red Cross
- Max Wyse for Christian Peacemaker Teams
- Liz Berg for Middle East Fellowship

►Everyone else can play vicariously.
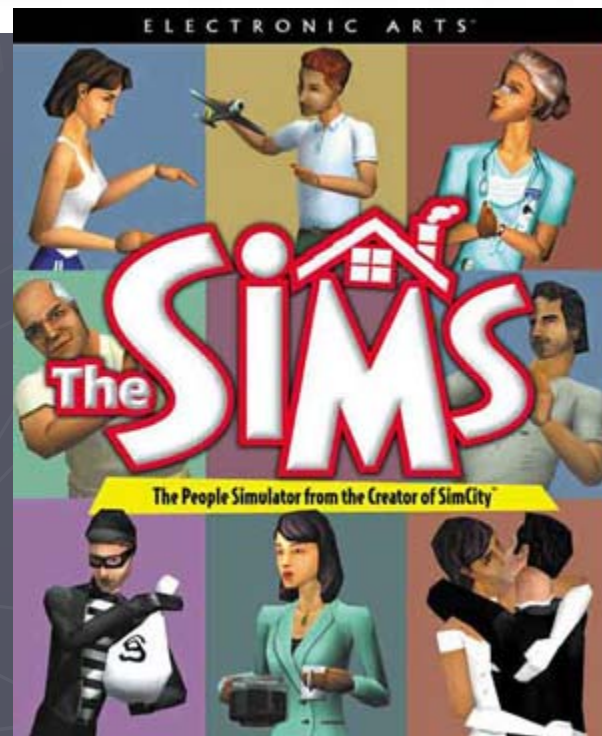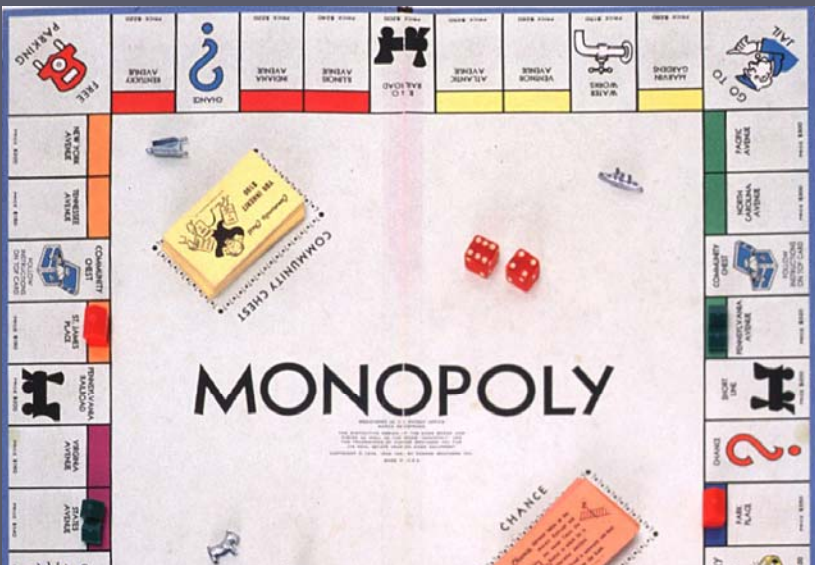
The Meek Will Inherit the Earth

► No communication among players during the game.

► Each player chooses an amount of money between $20 and $60.

  ▪ Example: $28.76.

► The choices are made in private and simultaneously on index cards or by clickers.

► The player who chooses the <u>smallest</u> amount will have that amount donated to her or his nonprofit organization.

► If there is a tie for the smallest amount, then that amount will be shared equally among the tying players.

The Meek Will Inherit the Earth

(players choosing the smallest amount share that amount)

21%	1.	$20
12%	2.	$25
11%	3.	$30
13%	4.	$35
14%	5.	$40
9%	6.	$45
3%	7.	$50
5%	8.	$55
11%	9.	$60

The Meek Will Inherit the Earth
(players choosing the smallest amount share that amount)

|		
17%	1.	$20
10%	2.	$25
6%	3.	$30
1%	4.	$35
4%	5.	$40
5%	6.	$45
5%	7.	$50
10%	8.	$55
41%	9.	$60
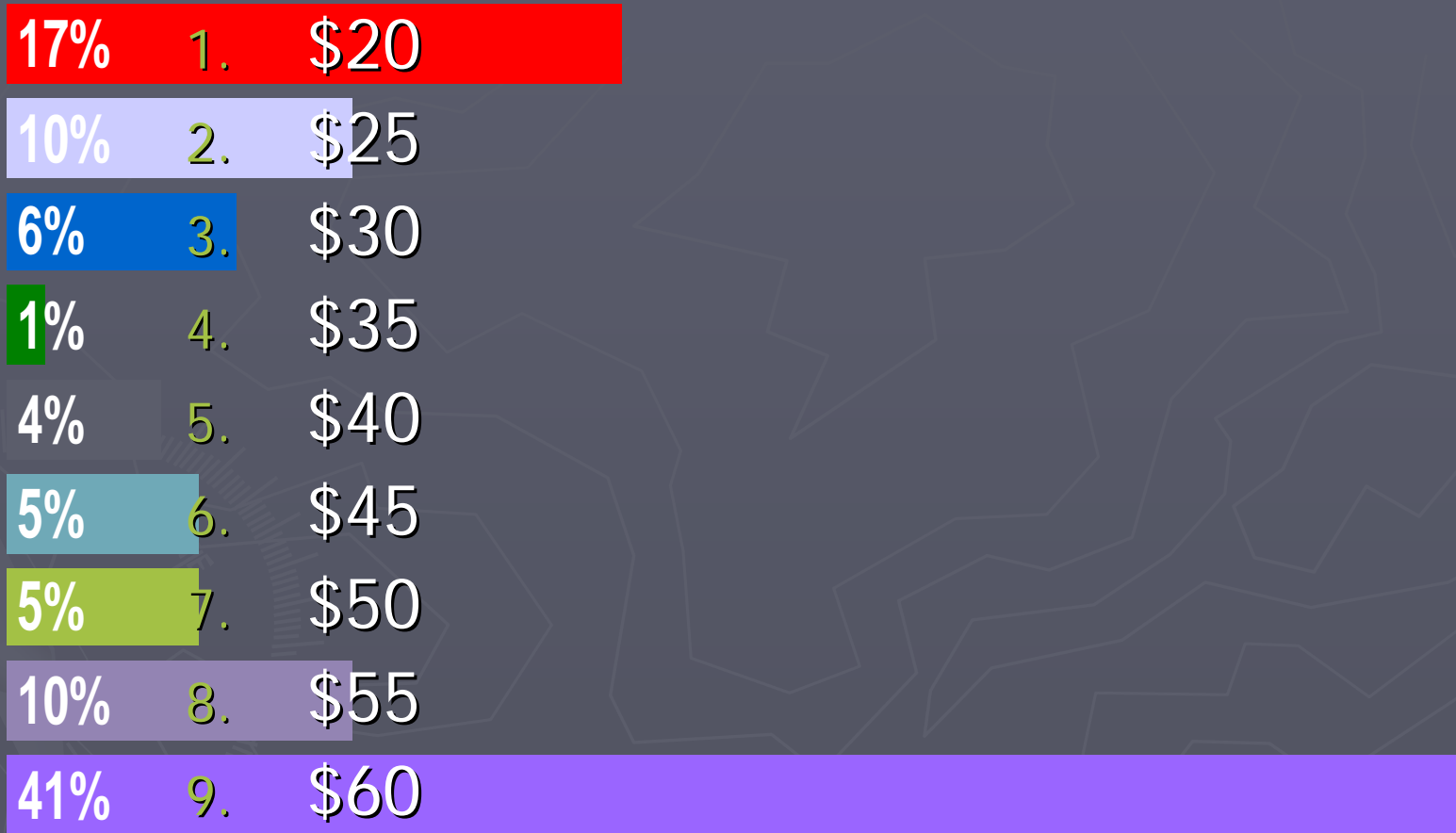
Mathematical Analysis

- ► Assumptions
  - Players are self-interested.
  - Players can foresee consequences of their actions.
  - These assumptions are common knowledge among the players.
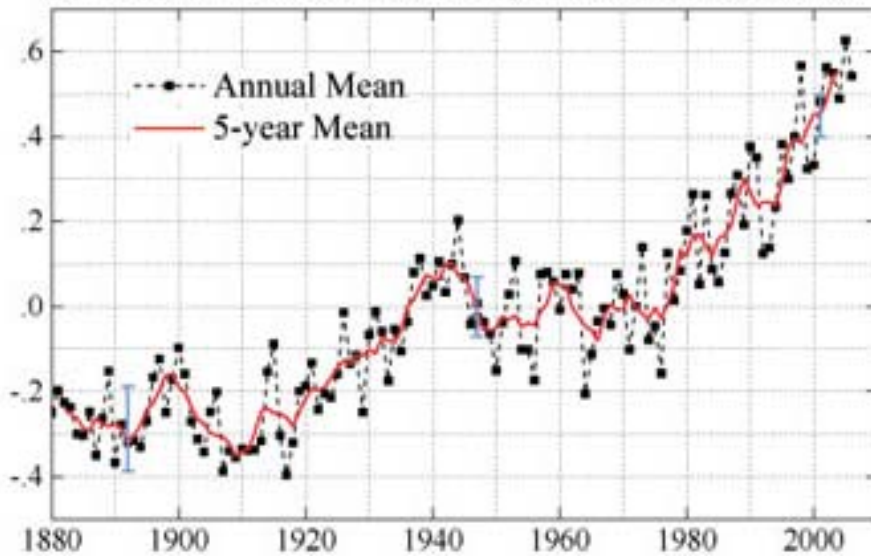- ► Conclusions
  - Is $30 the best choice?
    - ► No.
    - ► Neither is any other amount greater than $20.
  - Is $20 the best choice?
    - ► Yes.
    - ► Each player choosing $20 is the unique Nash equilibrium.
  - Would not $60 be better?
    - ► Yes, but …
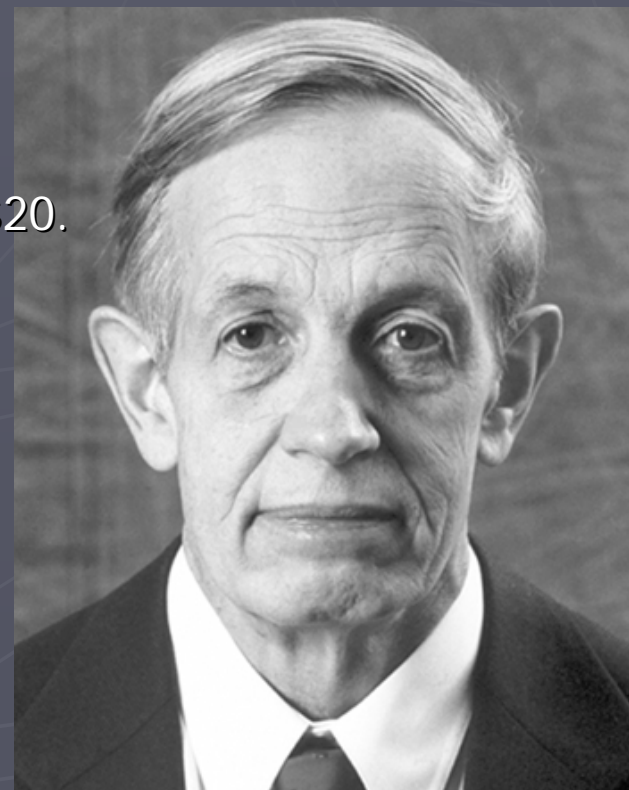
Real World

(a) Global-Mean Surface Temperature Anomaly (°C)

Ultimatum Game

► Proposer proposes how to divide $60 between herself or himself and another randomly selected player ($0 minimum).

  ▪ Example: $50 to me and $10 to you.

► Responder either accepts or rejects the proposal.

  ▪ Example: if offered at least $5, accept; otherwise, reject.

► Accepted proposal is carried out. Rejected proposal results in $0 to each player.

  ▪ Example: the Proposer receives $50 and the Responder receives $10.

► Everyone should choose a strategy for <u>each</u> role.

► The real players will be randomly matched and assigned roles.

Proposer's Action

9%   1. $60 for me, $0 for you

7%   2. $55 for me, $5 for you

2%   3. $50 for me, $10 for you

7%   4. $45 for me, $15 for you

11%   5. $40 for me, $20 for you

13%   6. $35 for me, $25 for you

45%   7. $30 for me, $30 for you

2%   8. $25 for me, $35 for you

**3%**   9. $20 for me, $40 for you

Responder's Action

22%	1.	Accept any offer
10%	2.	Accept an offer of $5 or more
4%	3.	Accept an offer of $10 or more
11%	4.	Accept an offer of $15 or more
10%	5.	Accept an offer of $20 or more
13%	6.	Accept an offer of $25 or more
28%	7.	Accept an offer of $30 or more
1%	8.	Accept an offer of $35 or more
2%	9.	Accept an offer of $40 or more

Theory, Experiments, and the Real World

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For his soul, my client wants $10 million, a seat in Congress, and Cindy Crawford.

It used to just take an apple.

► With self-interested players, the Responder should accept any positive offer and the Proposer should offer near 0.

► In experiments, modal and median offers are usually 40-50% and means are 30-40%.

► Some of this may be altruism but most of it is fear of rejection.

► In experiments, offers of 40-50% are rarely rejected and offers below 20% or so are rejected about half the time.

► Responders are willing to harm themselves over perceived unfair proposals.

MUSIC MARKET $6.98

Will you marry me? NO

Bibliography
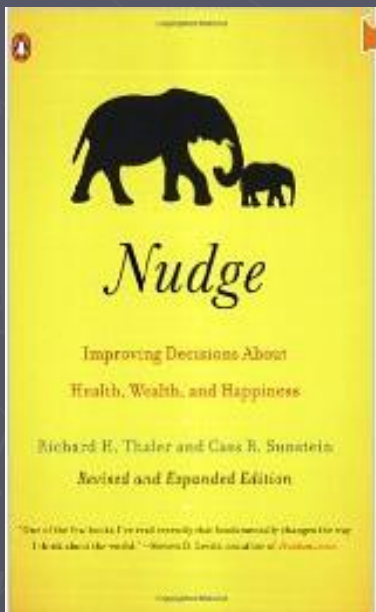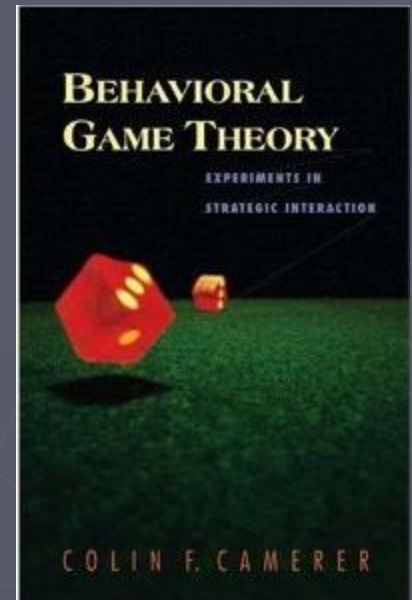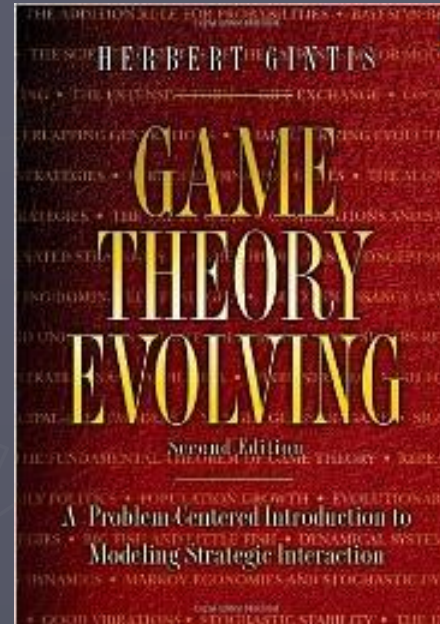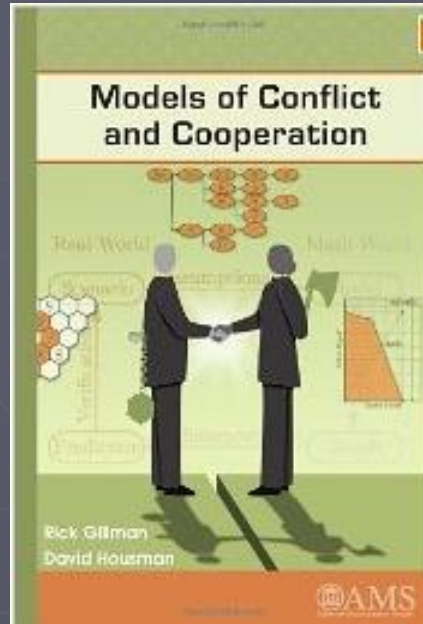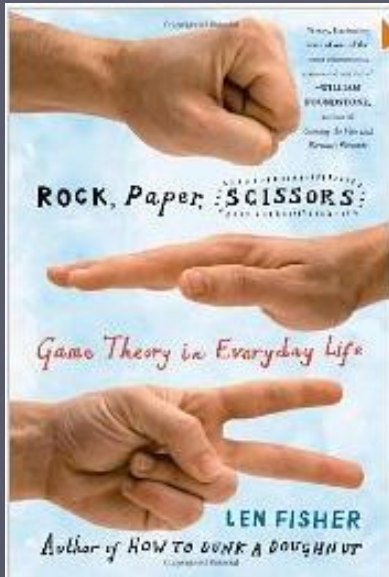
Math 250/350

Game Theory

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