Strategic Games, Theory, and Experiment

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Games, Theory, and Experiment Outline

- Introduction
- Ordinal Preferences
- Dictator Game
- Ultimatum Game
- Ah or Blee
- Beauty Contest
- A Strange Auction
- MARPS
- Conclusion

Ordinal Preferences

Dividing $4.00

Money to	
Self	Another
$0.00	$0.00
$0.00	$4.00
$0.50	$3.50
$1.00	$3.00
$1.50	$2.50
$2.00	$2.00
$2.50	$1.50
$3.00	$1.00
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$4.00	$0.00

- Imagine that you have a choice of ten possible outcomes.

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- Imagine that you have a choice of ten possible outcomes.
- For each outcome, you will be given some money and another randomly chosen person in this audience will be given some money.

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$1.50	$2.50	
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- If you could choose one of the ten outcomes, which would you choose? Give that outcome rank 1.

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- If you could choose one of the ten outcomes, which would you choose? Give that outcome rank 1.
- If you could choose one of the nine unranked outcomes, which would you choose? Give that outcome rank 2.
- Continue until all outcomes have been ranked.

Ordinal Preferences Results

Money to		Rank Motivation	
Self	Another	Self Other	Equity Self
$0.00	$0.00	10	2
$0.00	$4.00	9	10
$0.50	$3.50	8	8
$1.00	$3.00	7	6
$1.50	$2.50	6	4
$2.00	$2.00	5	1
$2.50	$1.50	4	3
$3.00	$1.00	3	5
$3.50	$0.50	2	7
$4.00	$0.00	1	9

- How many have ordinal preferences like those displayed?

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- What is the distribution of first-place ranks?
- What is the distribution of ranks for the outcome in which both persons receive $0.00?

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- Play now!

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- Over many experiments, dictators offer 20% on average. Social acceptance a factor.
- Many dictators offer 0% and very few offer more than 50%.

Ultimatum Game

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- Proposer: Of $4.00, I offer ________ to another person and will keep the rest.
- Responder: I will accept offers of ________ or greater.
- Randomly chosen audience members will be the Proposer and Responder.

- The Proposer chooses how to divide $4.00 with another person.
- The Responder decides whether to accept or reject the offer.
- If the proposal is accepted, money is distributed as proposed.
- If the proposal is rejected, no money is distributed.
- Play now!

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- In experiments, offers of 40-50% are rarely rejected and offers below 20% or so are rejected about half the time.
- So, self-interested players should make lower offers than found empirically.

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- Preschool children are self-interested, elementary school children are strict egalitarians, and middle school students are similar to adults.
- The average offer of University of Miami women students to attractive men was "hyperfair" (higher than 50%).
- In a project comparing a dozen simple societies in remote places such as Papua New Guinea, the Amazon basin, and Africa shows some societies close to the self-interested prediction and others with many "hyperfair" offers. Average offers are strongly correlated with the degree of "market integration."

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- There is no dilemma if
  - players are purely altruistic,
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  - there is repeated play.

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- There are n players, all players receive a for each player who chooses Ah, and Blee players receive an additional bonus $b > a$.

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- A player's expected payoff is

$$na + nap + nap^2 + \cdots = \frac{na}{1-p}$$

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- There is no incentive to deviate from the trigger strategy as long as

$$p > \frac{b-a}{(n-1)a}$$

Beauty Contest

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- Find the distribution of guesses as well as the winner.

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- Should we play the game again?

Dollar Auction

A Strange Auction

- Open ascending bid auction for a prize.

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- The highest bidder wins the prize but pays her bid.

A Strange Auction

- Open ascending bid auction for a prize.
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- The second highest bidder wins nothing but pays his bid.

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- Biological interpretation.

War of Attrition

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- Both of us pay for the war, but only one of us wins the prize.
- I know what the prize is worth to me but do not know what it is worth to you.

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- $f(v)$ is the probability density the prize is worth v to a player.

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- Assume $\beta(v)$ is the player's payoff maximizing bid, that is,

$$\pi(\beta(v)) \geq \pi(b)$$

for all $b \geq 0$.

War of Attrition Maximization (1)

- Maximize the following at $b = \beta(v)$:

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$$\pi'(b) = \frac{(v - \beta(\beta^{-1}(b)))f(\beta^{-1}(b))}{\beta'(\beta^{-1}(b)))} - (1 - F(\beta^{-1}(b))) + \frac{bf(\beta^{-1}(b))}{\beta'(\beta^{-1}(b)))}$$

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- Simplify.

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$$0 = vf(v)/\beta'(v) - (1 - F(v))$$

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which is differentiable and increasing where $f(v) > 0$.

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which is differentiable and increasing where $f(v) > 0$.

- Verify we have found a maximum by substituting back into formula for $\pi'(b)$.

$$\pi'(b) = (1 - F(\beta^{-1}(b))(v/\beta^{-1}(b) - 1)$$

which is positive if $b < \beta(v)$ and negative if $b > \beta(v)$.

War of Attrition Maximization (3)

- Optimal bidding strategy.

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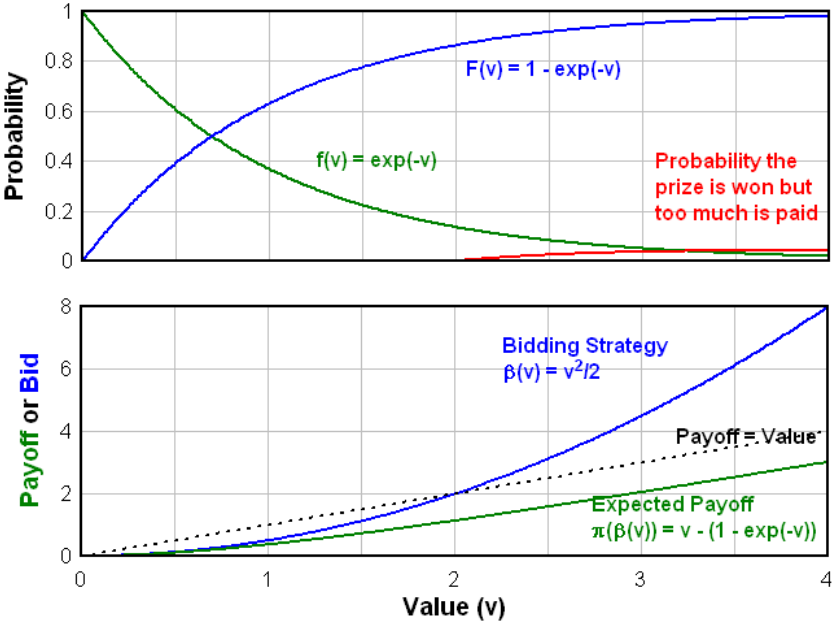
- For some prize values v, the bid $\beta(v)$ is greater than the value!



Probability
F(v) = 1 - exp(-v)
f(v) = exp(-v)
Probability the prize is won but too much is paid
Payoff or Bid
Bidding Strategy
β(v) = v²/2
Payoff = Value
Expected Payoff
π(β(v)) = v - (1 - exp(-v))
Value (v)

MARPS

Monetary Asymmetric Rock-Paper-Scissors

- You against everyone else.

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- Play now!

Repeated Monetary Asymmetric Rock-Paper-Scissors

- Two players.

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- Play it ten times with a single opponent now!

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- Biological interpretation.

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- Preference models are crucial.
- Experimental work is having a strong impact.
- There is a lot more for us to learn!

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Cooperative Games

David Housman

Goshen College

March 2008

Cooperative Games Outline

- Introduction
- Coalition Game: Dividing $6.00
- Bargaining Game: Time Share
- Fair Division Game: Bankruptcy
- Coalition Game: EPA
- Conclusion

Coalition Game: Dividing $6.00

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- Find a partner and decide who is player A and who is player B.

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- How many pairs were unable to come to an agreement?

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- How many pairs were unable to come to an agreement?
- Notation: $w(AB) = 6, w(A) = 1, w(B) = 3 \Longrightarrow x_A, x_B$?

Dividing \$6.00 Proportionately

$$x_i = \frac{w(i)}{w(A) + w(B)} w(AB)$$

- For $w(AB) = 6, w(A) = 1, w(B) = 3,$

$$x_A = \frac{1}{1+3} 6 = \$1.50, \quad x_B = \frac{3}{1+3} 6 = \$4.50$$

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- For $w(MH) = 6, w(M) = 4, w(H) = 0$,

$$x_M = \frac{4}{4+0} 6 = \$6, \quad x_H = \frac{0}{4+0} 6 = \$0$$

Dividing \$6.00 Equally

$x_i = \frac{1}{2}w(AB)$

- For $w(AB) = 6, w(A) = 1, w(B) = 3,$

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- How many pairs agreed upon this split?
- For the Microsoft/Housman partnership $w(MH) = 6, w(M) = 4, w(H) = 0,$
$$x_M = \frac{1}{2}6 = \$3, \quad x_H = \frac{1}{2}6 = \$3$$

Dividing the Surplus Equally

$x_i = w(i) + \frac{1}{2}(w(AB) - w(A) - w(B))$

- For $w(AB) = 6, w(A) = 1, w(B) = 3,$

$$x_A = 1 + \frac{1}{2}(6 - 1 - 3) = \$2.00, \quad x_B = 3 + \frac{1}{2}(6 - 1 - 3) = \$4.00$$

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- How many pairs agreed upon this split?
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$$x_M = 4 + \frac{1}{2}(6 - 4 - 0) = \$5, \quad x_H = 0 + \frac{1}{2}(6 - 4 - 0) = \$1$$

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- What were the other agreed upon splits?

Arbitration versus Negotiation

- Aristotle's Maxim: "Equals should be treated equally, and unequals unequally, in proportion to the relevant similarities, and differences."

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  - Reward: the child who worked hardest to fix and clean the flute.
  - Exogenous rights: the child whose father currently owns the flute.
  - Fitness: the child who plays the flute most beautifully.
- For $w(AB) = 6, w(A) = 1, w(B) = 3$,
  - A and B have exogenous rights to 1 and 3, respectively. But who should receive the surplus of 2?

Arbitration versus Negotiation

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  - Reward: the player who led the negotiation and wrote the contract.
  - Fitness: the player who will make better use of the money.
  - Compensation/Reward/Fitness: equal split if there is nothing to distinguish the two players.

Bargaining Game: Time Share

Time Share

- Ten employees work in an office and share a radio from which they can listen to stations that play rock, jazz, or country music. Five employees like rock and hate jazz and country. Three employees like jazz and hate rock and country. Two employees like country and hate rock and jazz. How should the employer allocate time across the three stations?

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		Payoffs									
Time	Employee	A	B	C	D	E	F	G	H	I	J
x	Rock	x	x	x	x	x	0	0	0	0	0
y	Jazz	0	0	0	0	0	y	y	y	0	0
z	Country	0	0	0	0	0	0	0	0	z	z

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Time	Employee	A	B	C	D	E	F	G	H	I	J
x	Rock	x	x	x	x	x	0	0	0	0	0
y	Jazz	0	0	0	0	0	y	y	y	0	0
z	Country	0	0	0	0	0	0	0	0	z	z

- Utilitarian: maximize payoff sum $(1, 0, 0)$.

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Time	Employee	A	B	C	D	E	F	G	H	I	J
x	Rock	x	x	x	x	x	0	0	0	0	0
y	Jazz	0	0	0	0	0	y	y	y	0	0
z	Country	0	0	0	0	0	0	0	0	z	z

- Utilitarian: maximize payoff sum $(1, 0, 0)$.
- Egalitarian: maximize the minimum payoff $(1/3, 1/3, 1/3)$.

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Time	Employee	A	B	C	D	E	F	G	H	I	J
x	Rock	x	x	x	x	x	0	0	0	0	0
y	Jazz	0	0	0	0	0	y	y	y	0	0
z	Country	0	0	0	0	0	0	0	0	z	z

- Utilitarian: maximize payoff sum $(1, 0, 0)$.
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- Nash: maximize the product of the payoffs $(5/10, 3/10, 2/10)$.

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Time	Employee	A	B	C	D	E	F	G	H	I	J
x	Rock	x	x	x	x	x	0	0	0	0	0
y	Jazz	0	0	0	0	0	y	y	y	0	0
z	Country	0	0	0	0	0	0	0	0	z	z

- Utilitarian: maximize payoff sum $(1, 0, 0)$.
- Egalitarian: maximize the minimum payoff $(1/3, 1/3, 1/3)$.
- Nash: maximize the product of the payoffs $(5/10, 3/10, 2/10)$.
- Lifeboat and Reviewer variations.

Fair Division Game: Bankruptcy

Bankruptcy

- Divide yourselves into groups of three, and decide who will be player A, player B, and player C.

Bankruptcy

- Divide yourselves into groups of three, and decide who will be player A, player B, and player C.
- The Housman Company has declared itself bankrupt. It has $6 in assets remaining. Players A, B, and C are owed $3, $6, and $9, respectively.

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- Your goal is to come to an agreement over how to divide $6 among the three of you.

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- If you cannot come to an agreement, then the $6 pays for legal fees and each player receives $0.

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- If you cannot come to an agreement, then the $6 pays for legal fees and each player receives $0.
- One group will be randomly chosen to receive the agreed upon split or the no agreement payments.

Bankruptcy

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- The agreement must be in writing and signed by all three players.

Bankruptcy

- Divide yourselves into groups of three, and decide who will be player A, player B, and player C.
- The Housman Company has declared itself bankrupt. It has \$6 in assets remaining. Players A, B, and C are owed \$3, \$6, and \$9, respectively.
- Your goal is to come to an agreement over how to divide \$6 among the three of you.
- If you cannot come to an agreement, then the \$6 pays for legal fees and each player receives \$0.
- One group will be randomly chosen to receive the agreed upon split or the no agreement payments.
- The agreement must be in writing and signed by all three players.
- Play now!

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00
Proportional Split	1.00	2.00	3.00

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00
Proportional Split	1.00	2.00	3.00
Equal Loss Split	0.00	1.50	4.50

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00
Proportional Split	1.00	2.00	3.00
Equal Loss Split	0.00	1.50	4.50
Talmudic Split	1.50	2.25	2.25

Bankruptcy Results

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Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00
Proportional Split	1.00	2.00	3.00
Equal Loss Split	0.00	1.50	4.50
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A mishna (a short statement of the law) from the Babylonan Talmud (a collection of Jewish religious and legal decisions set down during the first five centuries A.D.) . . .

Wife$\longrightarrow$	A	B	C
Assets$\downarrow$ Owed$\longrightarrow$	3.00	6.00	9.00

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00
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3.00	1.00	1.00	1.00

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Wife⟶	A	B	C	
Assets↓ Owed⟶	3.00	6.00	9.00	
3.00	1.00	1.00	1.00	Equal split?

Bankruptcy Results

Player	A	B	C
Owed	3.00	6.00	9.00
Equal Split	2.00	2.00	2.00
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3.00	1.00	1.00	1.00
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Equal split?

Bankruptcy Results

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Equal split?

What is this?

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Talmudic Law of Contracts

- Two hold a garment; one claims it all, the other claims half. What is the equitable division of the garment?

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- Equal Loss Split: Give 3/4 to the first and 1/4 to the second so that each has lost 1/4 of the garment.
- Talmudic Split: The second has conceded half to the first and the remaining half should be split equally, so 3/4 to the first and 1/4 to the second.

Talmudic Split Applied Consistently

Player	A	B	C
Assets↓ Owed⟶	3.00	6.00	9.00

Talmudic Split Applied Consistently

Player	A	B	C	
Assets↓ Owed⟶	3.00	6.00	9.00	
3.00	1.00	1.00	1.00	No concessions

Talmudic Split Applied Consistently

Player	A	B	C	
Assets↓ Owed⟶	3.00	6.00	9.00	
3.00	1.00	1.00	1.00	No concessions
6.00	1.50	2.25	2.25	A concedes 0.75 to B

Talmudic Split Applied Consistently

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6.00	1.50	2.25	2.25	A concedes 0.75 to B
9.00	1.50	3.00	4.50	A concedes 1.50 to B

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9.00	1.50	3.00	4.50	A concedes 1.50 to B

B concedes 1.50 to C

Talmudic Split Applied Consistently

Player	A	B	C	
Assets↓ Owed⟶	3.00	6.00	9.00	
3.00	1.00	1.00	1.00	No concessions
6.00	1.50	2.25	2.25	A concedes 0.75 to B
9.00	1.50	3.00	4.50	A concedes 1.50 to B
				B concedes 1.50 to C
6.00	x	y	z	

Talmudic Split Applied Consistently

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- Clearly, $x + y + z = 6$ and $0 \leq x \leq y \leq z$.

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- Clearly, $x + y + z = 6$ and $0 \leq x \leq y \leq z$.
- Since $y + z \leq 6$, neither B nor C concedes anything. Thus, $y = z$.

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6.00	x	y	z	

- Clearly, $x + y + z = 6$ and $0 \leq x \leq y \leq z$.
- Since $y + z \leq 6$, neither B nor C concedes anything. Thus, $y = z$.
- So, $x + 2y = 6$ implies $y = 3 - x/2$ implies $x + y = 3 + x/2$ implies A concedes $x/2$.

Talmudic Split Applied Consistently

Player	A	B	C	
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- Clearly, $x + y + z = 6$ and $0 \leq x \leq y \leq z$.
- Since $y + z \leq 6$, neither B nor C concedes anything. Thus, $y = z$.
- So, $x + 2y = 6$ implies $y = 3 - x/2$ implies $x + y = 3 + x/2$ implies A concedes $x/2$.
- Since B concedes nothing to A, $x = 3/2$ and $z = y = 3 - 3/4$.

Coalition Game: EPA Game

EPA Game

- Divide yourselves into groups of four, and decide who will be player A, player B, player C, and player D.

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- The Environmental Protection Agency has mandated improvements in the sewage treatment facilities in the cities of Avon, Barport, Claron, and Delmont. Each city could work separately, but $140 million would be saved by all four working together. If one of the cities was unwilling to cooperate, some other groups of cities could also save money as summarized in the table.

Coalition	ABCD	ABC	ABD	ACD	AB	any other
Savings	140	108	96	84	24	0

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Savings	140	108	96	84	24	0

- Your goal is to come to a written and signed agreement.

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Coalition	ABCD	ABC	ABD	ACD	AB	any other
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- Your goal is to come to a written and signed agreement.
- One group will be randomly chosen to receive the agreed upon amounts (divided by $10 million).
- Play now!

Shapley: Seeking Simplicity

Coalition	G1		G2		G3		G4		G5		EPA
ABCD	24		84		72		84		124		140
ABC	24		0		0		84		0		108
ABD	24		0		72		0		0		96
ACD	0		84		0		0		0		84
AB	24		0		0		0		0		24
anything else	0		0		0		0		0		0
Player	A1		A2		A3		A4		A5		EPA
A											
B											
C											
D											

Shapley: Efficient & Unbiased

Coalition	G1		G2		G3		G4		G5		EPA
ABCD	24		84		72		84		124		140
ABC	24		0		0		84		0		108
ABD	24		0		72		0		0		96
ACD	0		84		0		0		0		84
AB	24		0		0		0		0		24
anything else	0		0		0		0		0		0
Player	A1		A2		A3		A4		A5		EPA
A									31		
B									31		
C									31		
D									31		

Shapley: Efficient, Unbiased, & Subsidy Free

Coalition	G1		G2		G3		G4		G5		EPA
ABCD	24		84		72		84		124		140
ABC	24		0		0		84		0		108
ABD	24		0		72		0		0		96
ACD	0		84		0		0		0		84
AB	24		0		0		0		0		24
anything else	0		0		0		0		0		0
Player	A1		A2		A3		A4		A5		EPA
A	12		28		24		28		31		
B	12		0		24		28		31		
C	0		28		0		28		31		
D	0		28		24		0		31		

Shapley: Efficient, Unbiased, Subsidy Free, & Additive

Coalition	G1		G2		G3		G4		G5		EPA
ABCD	24	+	84	+	72	+	84	−	124	=	140
ABC	24	+	0	+	0	+	84	−	0	=	108
ABD	24	+	0	+	72	+	0	−	0	=	96
ACD	0	+	84	+	0	+	0	−	0	=	84
AB	24	+	0	+	0	+	0	−	0	=	24
anything else	0	+	0	+	0	+	0	−	0	=	0
Player	A1		A2		A3		A4		A5		EPA
A	12	+	28	+	24	+	28	−	31	=	61
B	12	+	0	+	24	+	28	−	31	=	33
C	0	+	28	+	0	+	28	−	31	=	25
D	0	+	28	+	24	+	0	−	31	=	21

Shapley: Not Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0

Player	Shapley
A	61
B	33
C	25
D	21

Shapley: Not Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0
Player	**Shapley**
A	61
B	33
C	25
D	21

Suppose C and D are satisfied but A and B want to renegotiate.

Shapley: Not Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0

Player	Shapley
A	61
B	33
C	25
D	21

Suppose C and D are satisfied but A and B want to renegotiate.

Coalition	EPA
AB	$61 + 33 = 94$
A	$84 - 25 - 21 = 38$
B	0

Shapley: Not Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0

Player	Shapley
A	61
B	33
C	25
D	21

Suppose C and D are satisfied but A and B want to renegotiate.

Coalition	EPA
AB	$61 + 33 = 94$
A	$84 - 25 - 21 = 38$
B	0

Player	Shapley
A	$38 + \frac{1}{2}56 = 68$
B	$0 + \frac{1}{2}56 = 28$

Shapley: Not Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0
Player	**Shapley**
A	61
B	33
C	25
D	21

Suppose C and D are satisfied but A and B want to renegotiate.

Coalition	EPA
AB	$61 + 33 = 94$
A	$84 - 25 - 21 = 38$
B	0
Player	**Shapley**
A	$38 + \frac{1}{2}56 = 68$
B	$0 + \frac{1}{2}56 = 28$

The renegotiation changes the recommended payoffs.

Nucleolus: Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0

Player	Nucleolus
A	a
B	b
C	c
D	d

Nucleolus: Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0
Player	**Nucleolus**
A	a
B	b
C	c
D	d

A and B renegotiation:

Coalition	Gain
AB	$a+b$
A	$84-c-d$
B	0

Nucleolus: Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0
Player	**Nucleolus**
A	a
B	b
C	c
D	d

A and B renegotiation:

Coalition	Gain
AB	$a+b$
A	$84-c-d$
B	0

Consistency requires

$$\begin{aligned} b &= 0+\frac{1}{2}(a+b-(84-c-d)-0) \\ 2b &= a+b+c+d-84 \\ b &= (140-84)/2 = 28 \end{aligned}$$

Nucleolus: Consistent with Renegotiation

Coalition	EPA
ABCD	140
ABC	108
ABD	96
ACD	84
AB	24
anything else	0
Player	**Nucleolus**
A	a
B	b
C	c
D	d

A and B renegotiation:

Coalition	Gain
AB	$a+b$
A	$84-c-d$
B	0

Consistency requires

$$\begin{aligned} b &= 0+\frac{1}{2}(a+b-(84-c-d)-0) \\ 2b &= a+b+c+d-84 \\ b &= (140-84)/2 = 28 \end{aligned}$$

Consistency of A&C and A&D renegotiations imply the nucleolus is $(74, 28, 22, 16)$.

You Can't Always Get What You Want (1)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Shapley	Shapley
A	61	56
B	33	28
C	25	20
D	21	16

You Can't Always Get What You Want (1)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Shapley	Shapley
A	61	56
B	33	28
C	25	20
D	21	16

- But $56 + 28 + 20 < 108$.

You Can't Always Get What You Want (1)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Shapley	Shapley
A	61	56
B	33	28
C	25	20
D	21	16

- But $56 + 28 + 20 < 108$.
- Shapley is not always coalition rational.

You Can't Always Get What You Want (2)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Nucleolus	Nucleolus
A	74	84
B	28	18
C	22	12
D	16	6

You Can't Always Get What You Want (2)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Nucleolus	Nucleolus
A	74	84
B	28	18
C	22	12
D	16	6

- But $74 < 84$.

You Can't Always Get What You Want (2)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Nucleolus	Nucleolus
A	74	84
B	28	18
C	22	12
D	16	6

- But $74 < 84$.
- Nucleolus is not always coalition monotone.

You Can't Always Get What You Want (2)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Nucleolus	Nucleolus
A	74	84
B	28	18
C	22	12
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- But $74 < 84$.
- Nucleolus is not always coalition monotone.
- Shapley is coalition monotone but not always coalition rational.

You Can't Always Get What You Want (2)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
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Player	Nucleolus	Nucleolus
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- But $74 < 84$.
- Nucleolus is not always coalition monotone.
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- Nucleolus is coalition rational but not always coalition monotone.

You Can't Always Get What You Want (2)

Coalition	EPA	EPA2
ABCD	140	120
ABC	108	108
ABD	96	96
ACD	84	84
AB	24	24
anything else	0	0
Player	Nucleolus	Nucleolus
A	74	84
B	28	18
C	22	12
D	16	6

- But $74 < 84$.
- Nucleolus is not always coalition monotone.
- Shapley is coalition monotone but not always coalition rational.
- Nucleolus is coalition rational but not always coalition monotone.
- **Theorem.** There is no allocation method that is always efficient, coalition rational, and coalition monotone.

Conclusions

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- Axiomatics is applied math!

Conclusions

- Games are fun!
- Axiomatics is applied math!
- There is a lot more for us to learn!

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