Research in Undergraduate Mathematics Education Conference

PROOF SCHEMES AND LEARNING STRATEGIES OF ABOVE-AVERAGE MATHEMATICS STUDENTS

Preliminary Results

Mary Porter, St. Mary's College
David Housman, Goshen College

Participants

- Eleven undergraduates who had received As and Bs in math courses
- Four students had no proof-based course (all 1st year). Three students had 1 proof-based course (all 2nd year). Four students had at least 2 proof-based courses (2nd, 3rd, and two 4th year)

Interviews

- For each of seven conjectures, the student provided a proof or disproof and discussed how convincing her argument was to herself, a peer, and a mathematician.
- The student learned a mathematical concept from its definition and discussed what and how she learned.

Proof Schemes Data Collection

For 40 minutes, students examined 7 conjectures, stated whether each conjecture was true or false, and provided proofs.

In a 20 minute videotaped interview, students were asked for each conjecture the following:

- How certain are you that the conjecture is true or false?
- How convincing is your proof to you?
- How convincing would your proof be to a peer?
- How convincing would your proof be to a mathematician?

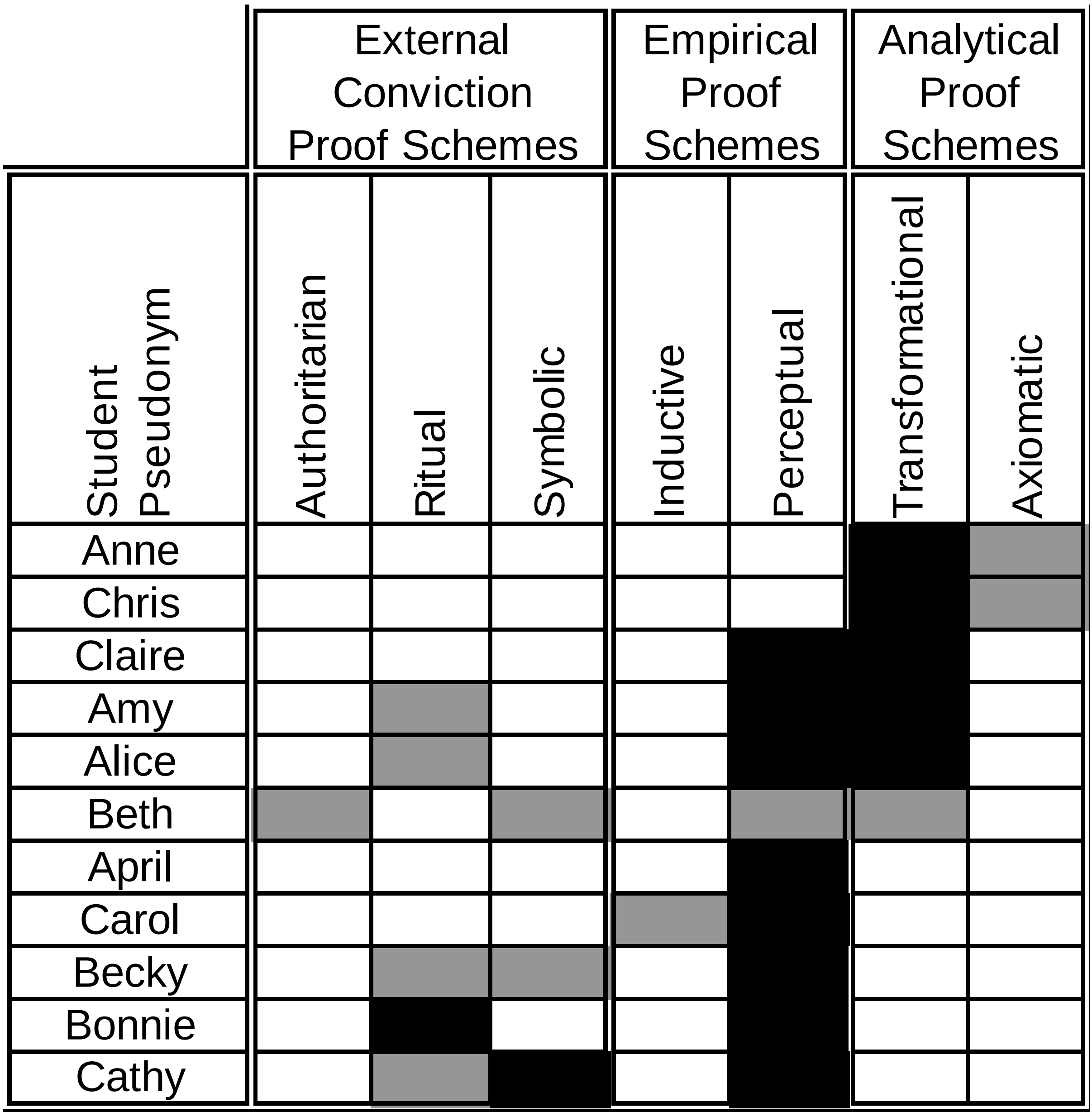
Proof Schemes Conjectures

1. The sum of the three interior angles of any triangle is 180 degrees.
2. If no angle of a quadrilateral is obtuse, then the quadrilateral is a rectangle.
3. If $(a + b)^2$ is even, then a and b are even.
4. The product of two negative real numbers is always a positive real number.
5. A polynomial of degree 3 must have at least one real root.
6. If A is a subset of C and B is a subset of C, then the union of A and B is a subset of C.
7. If an operation $*$ is commutative, then $*$ is associative.

Students' Proof Schemes

Guershon Harel and Larry Sowder

CBMS issues in Mathematics Education, 1998



primary proof scheme exhibited

significant aspects exhibited

Learning Strategies Data Collection

In a 40-60 minute videotaped interview, students were presented with a new mathematical concept and asked to answer a sequence of questions, both written and oral, involving the new concept.

Interviewer tried to neither confirm nor dispute assertions made by the student.

Instrument and analysis based on work of Randy Dahlberg and David Housman (1997), Facilitating Learning Events Through Example Generation, *Educational Studies in Mathematics*, volume 33, pages 283-299.

Learning Strategies Instrument

- Definition: A function is called *fine* if it has a root (zero) at each integer.
- Requests for an example, a non-example, and an explanation in student's own words and/or pictures.
- Determine whether six given functions are fine or not.

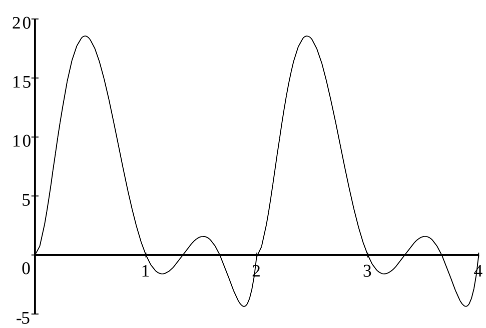
$$f(x) = \sin(\pi x)$$

$$f(x) = \tan\left(\tfrac{\pi}{2} x\right)$$

$$f(x) = x^2 - x$$

$$f(x) = 0$$

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$



Learning Strategies Instrument

- Determine whether four conjectures are true or not.

  1. No nonzero polynomial is a fine function.
  2. All trigonometric functions are fine.
  3. All fine functions are periodic.
  4. The product of a fine function and any other function is a fine function.

- Compare this learning experience with your usual approach.

Learning Strategies

A *learning strategy* is a method used by an individual to develop a concept image.

- Example Generation
- Example Usage
- Reformulation
- Assistance

Learning Events

A *learning event* has occurred when the student communicates a substantially new understanding of the concept.

Proof Schemes and Strategies

	External Conviction Proof Schemes			Empirical Proof Schemes		Analytical Proof Schemes		Strategy			
Student Pseudonym	Authoritarian	Ritual	Symbolic	Inductive	Perceptual	Transformational	Axiomatic	Example Generation	Example Usage	Early Reformulation	Total Reformulation
Carol				▒	■			■	▒	2.0	3.5
Becky		▒	▒		■			■	■	3.0	3.0
April					■			■	▒	0.5	3.5
Chris						■	▒	▒	■	1.5	2.5
Anne						■	▒	▒	■	2.0	3.0
Beth	▒		▒		▒	▒		▒	■	3.0	3.5
Amy		▒			■	■		▒	▒	1.0	2.0
Claire					■	■		▒	▒	1.0	2.0
Alice		▒			■	■		▒	▒	1.5	3.5
Cathy		▒	■		■					0.5	1.5
Bonnie		■			■					0.5	0.5

	Proof Scheme	Examples	Examples Used
■	primary	unprompted	often
▒	significant aspects	when needed	sometimes
	insignificant	when requested	never

Reformulation: Correct (1.0) and incomplete (0.5) among verbal, graphical, numerical, symbolic, and factoring types.