

**National Council of Teachers of
Mathematics
Research Pre-session
78th Annual Meeting**

**PROOF SCHEMES
AND
LEARNING STRATEGIES
OF
ABOVE-AVERAGE
MATHEMATICS STUDENTS**

Preliminary Results

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Participants

- Eleven undergraduates who had received As and Bs in math courses
- “Ann” had already taken a foundations of mathematics and one semester of abstract algebra
- “Cathy” was about to begin the foundations of mathematics course

Proof Schemes Data Collection

For 40 minutes, students examined 7 conjectures, stated whether each conjecture was true or false, and provided proofs.

In a 20 minute videotaped interview, students were asked for each conjecture the following:

- How certain are you that the conjecture is true or false?
- How convincing is your proof to you?
- How convincing would your proof be to a peer?
- How convincing would your proof be to a mathematician?

Proof Schemes Instrument: Conjectures

1. The sum of the three interior angles of any triangle is 180 degrees.
2. If no angle of a quadrilateral is obtuse, then the quadrilateral is a rectangle.
3. If $(a + b)^2$ is even, then a and b are even.
4. The product of two negative real numbers is always a positive real number.
5. A polynomial of degree 3 must have at least one real root.
6. If A is a subset of C and B is a subset of C , then the union of A and B is a subset of C .
7. If an operation $*$ is commutative, then $*$ is associative.

Students' Proof Schemes

Guershon Harel and Larry Sowder

CBMS issues in Mathematics Education, 1998

Ann's Proof Scheme Expressions

- Primarily Transformational
- Hints of Axiomatic
- Provides deductive proofs for four of the seven conjectures

Cathy's Proof Scheme Expressions

- Perceptual
- Symbolic
- Some Ritual Aspects
- The only proof she gave that we, as mathematicians, found convincing was a counter-example to “If $(a + b)^2$ is even, then a and b are even.” But she did not find her counter-example to be convincing.

Learning Strategies Data Collection

In a 40-60 minute videotaped interview, students were presented with a new mathematical concept and asked to answer a sequence of questions, both written and oral, involving the new concept.

Interviewer tried to neither confirm nor dispute assertions made by the student.

Instrument and analysis based on work of Randy Dahlberg and David Housman (1997), *Facilitating Learning Events Through Example Generation*, *Educational Studies in Mathematics*, volume 33, pages 283-299.

Learning Strategies Instrument

- Definition: A function is called *fine* if it has a root (zero) at each integer.
- Requests for an example, a non-example, and a description.
- Determine whether six given functions are fine or not.
- Determine whether four conjectures are true or not.
- Compare this learning experience with your usual approach.

Learning Strategies

A learning strategy is a method used by an individual to develop a concept image.

- Example Generation
- Example Usage
- Reformulation
- Assistance

Learning Events

A learning event has occurred when the student communicates a substantially new understanding of the concept.

Ann's Learning Strategies and Events

- Strong use of reformulation, example generation, and example usage.
- Three major learning events.

Cathy's Learning Strategies and Events

- Example usage and reformulation.
- Interviewer implicit suggestions to recheck the definition are not acted upon. She makes changes in her views about fine function without regard to the actual definition.

