Indiana Section
Mathematical Association of America
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WHAT CONVINCES ABOVE-AVERAGE MATHEMATICS STUDENTS?

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Participants

- Eleven undergraduates who had received As and Bs in math courses
- Group 0: 4 students had no proof-based course (all 1st year)
- Group 1: 3 students had 1 proof-based course (all 2nd year)
- Group 2^+: 4 students had at least 2 proof-based courses (2nd, 3rd, and two 4th year)

Data Collection

For 40 minutes, students examined 7 conjectures, stated whether each conjecture was true or false, and provided proofs.

In a 20 minute interview, students were asked for each conjecture the following:

- How certain are you that the conjecture is true or false?
- How convincing is your proof to you?
- How convincing would your proof be to a peer?
- How convincing would your proof be to a mathematician?

Conjectures

1. The sum of the three interior angles of any triangle is 180 degrees.
2. If no angle of a quadrilateral is obtuse, then the quadrilateral is a rectangle.
3. If $(a + b)^2$ is even, then a and b are even.
4. The product of two negative real numbers is always a positive real number.
5. A polynomial of degree 3 must have at least one real root.
6. If A is a subset of C and B is a subset of C, then the union of A and B is a subset of C.
7. If an operation $*$ is commutative, then $*$ is associative.

How Convincing Did Students Believe Their Proofs to Be?

20	Very convincing to all
9	Very convincing to self but not to a mathematician
12	Only somewhat convincing to self
36	Unconvincing

How Many Proofs Did the Researchers Find Convincing (With Caveats)?

7	Deductive proofs (2 considered only somewhat convincing by students)
6	Counter-examples (1 considered unconvincing by student)
3	Counter-arguments

Mathematical Correctness
versus
Student Perception

	All	Self	Some	None	Total
Correct Proof	6	0	1	1	8
Minor Mistakes	6	0	1	0	7
Substantial Progress	5	2	1	0	8
Minimal Progress	3	7	9	35	54
Total	20	9	12	36	77

Students' Proof Schemes

Guershon Harel and Larry Sowder

CBMS issues in Mathematics Education, 1998

Transformational Proof Scheme Expressions

- 6 students provided 7 proofs of true conjectures that were judged to be fully deductive with at most minor mistakes (2, 1, 3*).
  - 2 were considered flawless.
  - 5 expressed concern about the truth of or acceptability of using certain results in their proofs.
- 6 counter-examples given for false conjectures (2, 2, 2)

Axiomatic Proof Scheme Expressions

- One group 2^+ student understood the Non-Euclidean counter-example to Conjecture 1
- One group 1 student understood the Non-Euclidean counter-example to Conjecture 1 and used multiple nonstandard definitions when considering Conjecture 6
- No one made substantial progress on Conjecture 7 other than noting that the conjecture would be true if only the basic four operations (+, –, ×, ÷) were considered.

Counter-Arguments

- A group 0 student used two counter-arguments instead of counter-examples.
- A group 2+ student provided a substantial counter-argument instead of a counter-example to Conjecture 3.
- Another group 0 student used a counter-argument in addition to a counter-example because she was unsure what interviewer was looking for.

Perceptual Proof Scheme Expressions

- only special cases examined
- expression of conjecture in informal language considered a proof

Inductive Proof Scheme Expressions

- One group 0 student used examples to justify
- 1^{st} and 2^{nd} year students used examples to illustrate, but not justify, some conjectures

Ritual Proof Scheme Expressions

- 5 students found at least one proof would be less convincing to a mathematician than to themselves because of the form (3, 1, 1)
- 2 students found at least one proof very convincing to themselves that involved serious framework errors (1, 0, 1)
- 5 students found at least one proof would be less convincing to others than to themselves because of insufficient clarity (1, 2, 2)
- 4 students found at least one proof would be less convincing to a mathematician than to themselves because of the results used (0, 2, 2)

Why A Proof Is Less Convincing To Mathematicians

- I haven’t done proof in a long time, so I didn’t quite remember the format. I just kind of wrote down what I thought.
- It needs to be a little more technical.
- I don’t really know quite what’s acceptable in writing a proof.
- I’ve forgotten how to go through and like the exact mathematical whatever I was supposed to do.
- People are mostly looking like for a ‘proof’ proof, and this is more like a ‘feeling’ proof . . . . A ‘feeling’ proof is like you just go by what you know, but there’s no like concrete like mathematical terms.
- I guess because I think it’s too short.

Symbolic Proof Scheme Expressions

- 8 instances of nonstandard or incorrect symbolic notation that was considered at least somewhat convincing (4, 3, 1)

- One group 0 student avoided symbolic notation completely

Authoritarian Proof Scheme Expressions

- 8 students made explicit appeals to authority, including purported rules, when discussing Conjectures 1 and 4 (4, 2, 2)
- 10 students were very certain of the truth of some conjecture without being able to exhibit any proof even somewhat convincing to themselves (3, 3, 4)
- 2 students were more sure of the truth of Conjecture 1 than Conjecture 2 even though no proof was given for Conjecture 1 and a "very convincing" proof was given for Conjecture 2 (0, 0, 2)

Explicit Appeals To Authority

- I said true because I remember from like sixth grade, you say you add up all the angles to 180°.
- Because this is something that I have just known. That I’ve been told going all the way back to junior high that this is just something that I’ve just accepted. I haven’t really thought much about trying to prove it.
- I know, because that’s in a textbook.
- I don’t think I’ve ever seen a proof of that. It kind of goes along with the ah, when they tell you, you agree.
- So if we define it as that, then what is there to prove from that?
- I knew if all of them were true or false, I just didn’t know how to prove them.