Kernel: SageMath 9.7

2.7

Eigenvectors / Eigenvalues of the $\hat{S}_{\mathbf{n}}$ operator.

In [1]:
```
var('t p')
Sn=matrix([[cos(t),sin(t)*exp(-I*p)],
          [sin(t)*exp(I*p),-cos(t)]])
print("The S_n operator in matrix form")
show(Sn)
```

Out[1]: The S_n operator in matrix form

$$\begin{pmatrix} \cos(t) & e^{(-i\,p)}\sin(t) \\ e^{(i\,p)}\sin(t) & -\cos(t) \end{pmatrix}$$

In [2]:
```
Sn.eigenvectors_right()
```

Out[2]:
```
[(-sqrt(cos(t)^2 + sin(t)^2),
  [(1, -(cos(t)*e^(I*p) + sqrt(cos(t)^2 + sin(t)^2)*e^(I*p))/sin(t))],
  1),
 (sqrt(cos(t)^2 + sin(t)^2),
  [(1, -(cos(t)*e^(I*p) - sqrt(cos(t)^2 + sin(t)^2)*e^(I*p))/sin(t))],
  1)]
```

Notice that the *first* one above is the one with the eigenvalue `-sqrt(cos(t)^2 + sin(t)^2)` $= -1$. So that must be the *spin down* eigenstate.
Copying the contents of the eigenvectors into two column vectors, and `show()`ing them makes it easier to digest...

In [6]:
```
edown=matrix([[1],[-(cos(t)*e^(I*p) + sqrt(cos(t)^2 +
sin(t)^2)*e^(I*p))/sin(t)]])
eup=matrix([[1],[ -(cos(t)*e^(I*p) - sqrt(cos(t)^2 +
sin(t)^2)*e^(I*p))/sin(t)]])
print("The spin DOWN eigenstate:")
show(edown)
print("The spin UP eigenstate:")
show(eup)
```

Out[6]: The spin DOWN eigenstate:

$$\begin{pmatrix} 1 \\ -\frac{\cos(t)e^{(i\,p)}+\sqrt{\cos(t)^2+\sin(t)^2}e^{(i\,p)}}{\sin(t)} \end{pmatrix}$$

The spin UP eigenstate:

$$\begin{pmatrix} 1 \\ -\frac{\cos(t)e^{(i\,p)}-\sqrt{\cos(t)^2+\sin(t)^2}e^{(i\,p)}}{\sin(t)} \end{pmatrix}$$

Ah-ha, we can do two things to simplify this a bit further:

- Pull out the common factor of $e^{(ip)}$ (where 'p' stands for ϕ)

- Recognize the trig identity, that the square root is just **1**.

So, to summarize...

In [7]:

```
edown=matrix([[1],[-e^(I*p)*(cos(t) + 1)/sin(t)]])
eup=matrix([[1],[ -e^(I*p)*(cos(t) - 1)/sin(t)]])
print("The spin DOWN eigenstate:")
show(edown)
print("The spin UP eigenstate:")
show(eup)
```

Out[7]:

```
The spin DOWN eigenstate:
```

$$\left(\begin{array}{r} 1 \\ -\frac{(\cos(t)+1)e^{(i\,p)}}{\sin(t)} \end{array}\right)$$

```
The spin UP eigenstate:
```

$$\left(\begin{array}{r} 1 \\ -\frac{(\cos(t)-1)e^{(i\,p)}}{\sin(t)} \end{array}\right)$$

Test that they're eigenvectors: Since 1 is the eigenvalue of the 'up' state, test if $S_n e_\uparrow \stackrel{?}{=} 1e_\uparrow$.

In [9]:

```
show(Sn*eup == eup) #true or false?
```

Out[9]: True

What about $S_n e_\downarrow \stackrel{?}{=} -1e_\downarrow$

In [10]:

```
show(Sn*edown == -1*edown)
```

Out[10]: True

In [0]: