Kernel: SageMath 9.7

1.10

Here are the column-vector representations of $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$ in the z basis.

In [46]:
```
ket1=matrix( [[4/5],[3*I/5]])
ket2=matrix( [[4/5],[-3*I/5]])
ket3=matrix( [[-4/5],[3*I/5]])

show(ket1,ket2,ket3)
```

Out[46]:
$$\begin{pmatrix} \frac{4}{5} \\ \frac{3}{5}i \end{pmatrix} \begin{pmatrix} \frac{4}{5} \\ -\frac{3}{5}i \end{pmatrix} \begin{pmatrix} -\frac{4}{5} \\ \frac{3}{5}i \end{pmatrix}$$

In [0]:
```
```

In [0]:
```
```

Now we need the bras $\langle +|$, ${}_x\langle +|$ and ${}_y\langle +|$. Each is the *complex conjugate* of the transpose of the corresponding ket. So, for ${}_y\langle +|$ this means...

In [4]:
```
up_y_ket=1/sqrt(2)*matrix([[1],[I]])
print("ket spin up_y = ")
show(up_y_ket)
bray=conjugate(transpose(up_y_ket))
print("But, bra spin up_y = ")
show(bray)
```

Out[4]:
```
ket spin up_y =
```
$$\begin{pmatrix} \frac{1}{2}\sqrt{2} \\ \frac{1}{2}i\sqrt{2} \end{pmatrix}$$
```
But, bra spin up_y =
```
$$\begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}i\sqrt{2} \end{pmatrix}$$

In [5]:
```
print("Here are all 3 bra's")
braz=matrix( [[1,0]])
brax=(1/sqrt(2))*matrix( [[1,1]] )
# Remember that bra psi is the transpose, complex conjugate of ke
show(braz, brax, bray)
```

Out[5]:
```
Here are all 3 bra's
```
$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}i\sqrt{2} \end{pmatrix}$$

I'll only calculate the probabilities of measuring spin up in the z, x, and y orientations...

In [56]:
```
#'ip' stands for inner product of the ket with spin up_z, spin up_x,
and spin up_y
ipz=braz*ket1
ipx=brax*ket1
ipy=bray*ket1
print("PROBABILITIES for ket 1")
print("P_+ =",ipz*conjugate(ipz))
print("P_+x=",ipx*conjugate(ipx))
print("P_+y=",ipy*conjugate(ipy))
```

Out[56]:
```
PROBABILITIES for ket 1
P_+ = [16/25]
P_+x= [1/2]
P_+y= [49/50]
```

In [57]:
```
ipz=braz*ket2
ipx=brax*ket2
ipy=bray*ket2
print("PROBABILITIES for ket 2")
print("P_+ =",ipz*conjugate(ipz))
print("P_+x=",ipx*conjugate(ipx))
print("P_+y=",ipy*conjugate(ipy))
```

Out[57]:
```
PROBABILITIES for ket 2
P_+ = [16/25]
P_+x= [1/2]
P_+y= [1/50]
```

In [58]:
```
ipz=braz*ket3
ipx=brax*ket3
ipy=bray*ket3
print("PROBABILITIES for ket 3")
print("P_+ =",ipz*conjugate(ipz))
print("P_+x=",ipx*conjugate(ipx))
print("P_+y=",ipy*conjugate(ipy))
```

Out[58]:
```
PROBABILITIES for ket 3
P_+ = [16/25]
P_+x= [1/2]
P_+y= [1/50]
```

In [0]: