Lagrange Multiplier Problems

#1: $f(x, y) = x^2 - 8x + y^2 - 12y + 48$, and constraint $x + y = 8$

```
myf = x^2 - 8 x + y^2 - 12 y + 48;
myg = x + y;
constraint = myg == 8;
Grad[myf, {x, y}]
Grad[myg, {x, y}]
```

```
{-8 + 2 x, -12 + 2 y}
```

```
{1, 1}
```

```
Solve[
 -8 + 2 x == λ * 1 &&
  -12 + 2 y == λ * 1 &&
  constraint,
 {x, y, λ}
]
```

```
{{x → 3, y → 5, λ → -2}}
```

One critical point at (3,5).
To check for a min/max, we look at the second derivatives...

```
D[myf, {x, 2}]
D[myf, {y, 2}]
```

2

2

Both 2nd derivatives are positive, so this is a minimum, with value...

```
myf /. {x → 3, y → 5}
```

-2

#2: Minimum value of $f(x, y, z) = 2x^2 + y^2 + 3z^2$, and constraints $2x - 3y - 4z = 49$.

```
myf = 2 x^2 + y^2 + 3 z^2;
myg  =  2 x - 3 y - 4 z;
constraint = myg == 49;
Grad[myf, {x, y, z}]
Grad[myg, {x, y, z}]
```

```
{4 x, 2 y, 6 z}
```

```
{2, -3, -4}
```

```
Solve[
 4 x ==  λ * 2 &&
  2 y == λ * (-3)  &&
  6 z  ==  λ (-4) &&
  constraint,
 {x, y, z, λ}
]
```

```
{{x → 3, y → -9, z → -4, λ → 6}}
```

One critical point at (3, -9, -4).
To check for a min/max, we look at the second derivatives...

The value of the function at this point is...

```
myf /. {x → 3, y → -9, z → -4}
```

```
147
```

#3: Maximum value of $f(x, y, z) = x\,y + y\,z$, and 2 constraints, $x + 2\,y = 6$ and $x - 3\,z = 0$.

```
myf = x y + y z; myg = x + 2 y ; myh = x - 3 z;
Grad[myf, {x, y, z}]
Grad[myg, {x, y, z}]
Grad[myh, {x, y, z}]
```

```
{y, x + z, y}

{1, 2, 0}

{1, 0, -3}
```

```
Solve[
 y == λ * 1 + μ * 1 &&
  x + z == λ * 2 &&
  y == μ * (-3) &&
  myg == 6 && myh == 0,
 {x, y, z, λ, μ}
]
```

$$\left\{\left\{x \to 3,\ y \to \frac{3}{2},\ z \to 1,\ \lambda \to 2,\ \mu \to -\frac{1}{2}\right\}\right\}$$

One critical point at (3, 3/2, 1).
Looking at the the second derivatives...

```
D[myf, {x, 2}]
D[myf, {y, 2}]
D[myf, {z, 2}]
```

```
0

0

0
```

Interesting! at any rate, the value of the function at the critical point is...

```
myf /. {x → 3, y → 3 / 2, z → 1}
```

```
6
```

#4: Maximum value of $f(x, y, z) = x\,y\,z$,

and constraint, $x + y + z - 6 = 0$.

```
myf = x y z; myg = x + y + z - 6;
Grad[myf, {x, y, z}]
Grad[myg, {x, y, z}]
```

```
{y z, x z, x y}
```

```
{1, 1, 1}
```

```
Solve[
 y z == λ * 1 &&
  x z == λ * 1 &&
  x y == λ * 1 &&
  myg == 0,
 {x, y, z, λ}
]
```

```
{{x → 0, y → 0, z → 6, λ → 0}, {x → 0, y → 6, z → 0, λ → 0},
 {x → 2, y → 2, z → 2, λ → 4}, {x → 6, y → 0, z → 0, λ → 0}}
```

The point (0,0,0) is a minimum, The critical point that might be a maximum is at (2,2,2).
To check for a min/max, we look at the second derivatives...

The value of the function at the critical point is...

```
myf /. {x → 2, y → 2, z → 2}
```

```
8
```