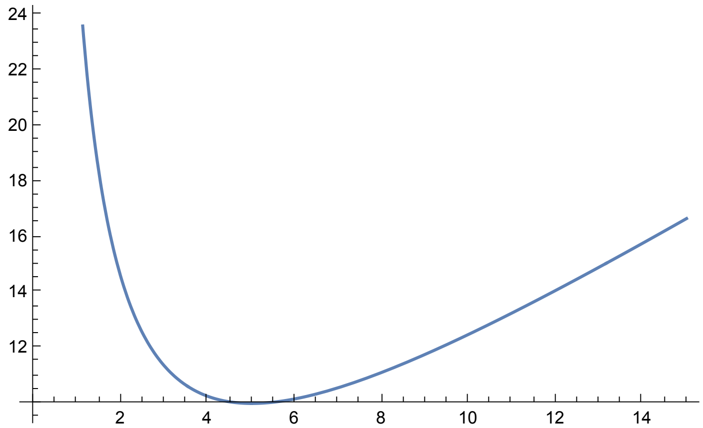
```
Clear["Global`*"]
```

```
U := u0 * (r / R + λ^2 * R / r)
```

What does it look like?? First term is a line with slope 1/R, second term is ~1/r, exploding as r->0:

```
Plot[U /. {u0 → 1, R → 1, λ → 5}, {r, 0.0001, 15}]
```



Position of minimum occurs where dU/dr = 0

```
D[U, r]
```

$$u0\left(\frac{1}{R}-\frac{R\,\lambda^2}{r^2}\right)$$

solve (1/R–Rλ^2/r^2 = 0) for r

```
Solve[R^(-1) - (R * λ^2) / r^2 == 0, r]
```

$\{\{r \to -R\,\lambda\}, \{r \to R\,\lambda\}\}$

We'll just take the positive solution, and call this r0. For the values of the graph above, this should be r0=5. Looks plausible.

```
r0 = R * λ
```

$R\,\lambda$

The effective spring constant, *k*, is the curvature (second derivative) evaluated at the position of the minimum.

```
D[D[U, r], r]
```

$$\frac{2\,R\,u0\,\lambda^2}{r^3}$$

```
k = % /. r → r0
```

$$\frac{2\,u0}{R^2\,\lambda}$$

The angular frequency of oscillation is $\omega=\sqrt{\frac{k}{m}}$, that is ....

```
Sqrt[k / m]
```

$\sqrt{2}\sqrt{\frac{u0}{m R^2 \lambda}}$

```
Ub = U /. {u0 → 1, R → 1, λ → 5, r → 5}
```

10

```
kb = k /. {u0 → 1, R → 1, λ → 5}
```

$\frac{2}{5}$

```
Ub + 0.5 kb * (r - 5) ^2
```

$10 + 0.2 (-5 + r)^2$

```
Plot[{U /. {u0 → 1, R → 1, λ → 5}, 10 + 1/2 kb * (r - 5) ^2}, {r, 4, 6}]
```