Math 211, Exam 2 Review

1. Suppose a function is given by a table of values as follows:

x	1.1	1.3	1.5	1.7	1.9	2.1
$f(x)$	12	15	21	23	24	25

(a) Estimate the instantaneous rate of change of f at $x = 1.7$.

(b) Use your answer in (b) to predict a value for f at $x = 1.8$.

(c) Is your prediction too large or too small? Explain.

2. Let $f(T)$ be the time, in minutes, that it takes for an oven to heat up to temperature T °F.

(a) Give the meaning, in plain English, of $f(300) = 10$.

It takes 10 minutes for the oven to heat up to 300 °F.

(b) What are the units of $f'(T)$?

f has units of minutes. T is °F. So $f' = df/dT$ has units of minutes/°F.

(c) Do you think $f'(T)$ would be positive or negative?

As the oven temperature increases, I think the time it takes (f) the oven to reach that temperature will increase. So I think $f'(T)$ is **positive.**

(d) Give the meaning, in plain English, of $f'(300) = 0.1$

For every additional degree above 300 °F, the oven will need an additional 0.1 minute.

3. A sports car accelerates from 0 ft/sec to 88 ft/sec in 5 seconds (88 ft/sec = 60 mph) The car's velocity is given in the table below.

t	0	1	2	3	4	5
$V(t)$	0	30	52	68	80	88

Find upper and lower bounds for the distance the car travels in 5 seconds.

A left sum will give an under estimate: (0 + 30 + 52 + 68 +80)*1=230 ft
A right sum will give an over estimate: (30+52+68+80+88)*1=318 ft

4. Let $f(t) = t^3 + t$.
   (a) What is the total change in $f(t)$ between $t = 2$ and $t = 5$?

   The change in *f* is *f(5)-f(2)=130-10=**120*** (I call this Δf)

   (b) What is the *average rate of change* in $f(t)$ between $t = 2$ and $t = 5$?

   The average rate of change is the total change in the function divided by the total change in x. That is $\Delta f/\Delta x = 120/(5-2) = 120/3 = \mathbf{40}$.

5. The flow rate of water in a mountain stream due to spring runoff is given in the following table. Give your ***best*** estimate for the total volume of water from 6:00 pm to midnight.

time (hours since 6:00 pm)	0	1	2	3	4	5	6
flow rate (in cubic meters per hour)	300	360	410	455	490	520	545

I'll do a left-sum and a right-sum and average them: left sum=2535; right sum=2780; average = 2657.5 or about 2658.

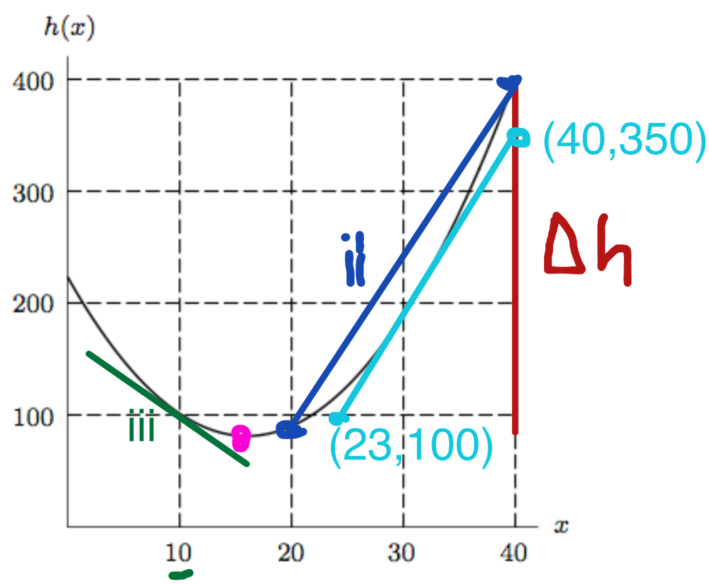
6. The graph of $h(x)$ is given to the right.
   (a) Draw on the graph (label your drawings and use different colors if you can)

   (i) A line segment whose length equals the change Δh in $h(x)$ between $x = 20$ and $x = 40$.

   (ii) A line segment whose slope equals the average rate of change $\frac{\Delta h}{\Delta x}$ of $h(x)$ between $x = 20$ and $x = 40$.

   (iii) A line whose slope equals the derivative $h'(10)$.

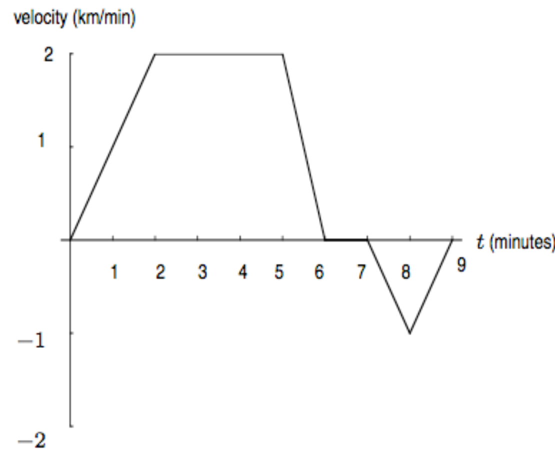
   (iv) A point on the graph where $h' = 0$.



   (b) Carefully estimate $h'(30)$

slope of the tangent line is
(350-100)/(40-23)=250/17 = 15 ish

7. A car is moving along a straight road from A to B starting from A at time $t = 0$.



To the right is the velocity (in km/min) plotted against time (in min).

How many kilometers away from A is the car at time

(b) $t = 2$
Area under v(t) between 0 and 2 is a triangle of height 2, base 2. So its area is 4/2=**2 km**

(c) $t = 5$ 2+3*2= **8 km**

(d) $t = 6$ 2 + 6+ 1= **9 km**

(e) $t = 7$ *v=0* between 6 and 7, so no change of distance: 9 km

(f) $t = 9$ 2 + 6 + 1 -1 = **8 km**

Find the derivatives of the following functions. Do not simplify.

(a) $f(x) = \sqrt{x}$

(b) $y = r^2 + 7r - 17$

(c) $h(t) = t^{\pi} + \sqrt{2}\, t$

(d) $g(x) = 2e^{\pi x}$

a.) $\sqrt{x} = x^{1/2}$

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D[√x, x]
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$\frac{1}{2\sqrt{x}}$

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D[x^(1/2), x]
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$\frac{1}{2\sqrt{x}}$

b.)

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D[r^2 + 7 r - 17, r]
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$7 + 2r$

c.)

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D[t^π + √2 t, t]
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$\sqrt{2} + \pi t^{-1+\pi}$

d.)

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D[2 e^(π x), x]
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$2 e^{\pi x} \pi$

(e) $y = \ln\left(x^3 + 4\right)$

e)

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D[Log[x^3 + 4], x]
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$$\frac{3\,x^2}{4 + x^3}$$

(f) $h(z) = z\cos\left(3z\right)$

f)

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D[z Cos[3 z], z]
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Cos[3 z] - 3 z Sin[3 z]
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(g) $f\left(x\right) = \dfrac{\ln x + 5}{x^2 + 7}$

g)

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D[ (Log[x] + 5) / (x^2 + 7), x]
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$$\frac{1}{x\left(7 + x^2\right)} - \frac{2\,x\,(5 + \text{Log}[x])}{\left(7 + x^2\right)^2}$$

8. The temperature, Y, in degrees Fahrenheit of a yam in a hot oven t minutes after it is placed there is given by

$$Y(t) = 350\,(1 - 0.7e^{-0.008\,t})$$

(a) What was the temperature of the yam when it was placed in the oven?

At $t=0$, $Y(0)=350(1-0.7)=350(0.3)=$ **105 degrees F.**

(b) If the yam is left on in the oven for a long time, it will eventually reach the temperature of the oven. What is the temperature of the oven?

$e^{-[\text{big\_number}]} = 0$. So Y(big number) $=350(1-0)=$**350 Degrees F is the temperature of the oven.**

When does the yam reach 175 ° F?

Solve $175 = 350\,(1 - 0.7e^{-0.008\,t})$ for t. I get…

at 42.06 minutes.

(c) What is $Y(20)$? What is $Y\,'(20)$? What do these quantities tell us about the temperature of the yam?

$Y(20)=141.2$; $Y'(t)= 0.7e^{-0.008}$; $Y'(20)=1.67$.
This means that at 20 minutes the yam's temperature is 141 degrees F, and the rate of change of its temperature is 1.67 degrees F per minute.

See below for a better calculation of Y'(t):

Let's say that the temperature of the yam is $Y(t) = 350(1 - 0.7e^{-0.008t})$ where the temperature is in degrees fahrenheit (F) and time is in minutes. 2

The derivative, $Y'(t)$ is the *rate of change of temperature with respect to time*, so it has units of degrees F / minute.

To calculate the derivative:

$$\begin{aligned}
Y'(t) &= \left[350(1 - 0.7e^{-0.008t})\right]' \\
&= 350\left[(1 - 0.7e^{-0.008t})\right]' \\
&= 350[1]' - 350\left[0.7e^{-0.008t}\right]' \\
&= 350 \cdot 0 - 350 \cdot 0.7\left[e^{-0.008t})\right]' \\
&= 0 - 350 \cdot 0.7 \cdot (-0.008) \cdot e^{-0.008t} \\
&= +350 \cdot 0.7 \cdot (0.008)e^{-0.008t}
\end{aligned}$$

Evaluating $Y'(20) = 350 \cdot 0.7 \cdot 0.008e^{-0.008 \cdot 20}$:

In [1]: 3
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350*0.7*0.008*e^(-0.008*20)
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Out[1]:
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1.67020182637377
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That's 1.67 degrees F / minute. 4