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FINDING A VALUE ON PARTIALLY DEFINED GAMES

Introduction

A cooperative game is a pair (N, v) where N = {1, 2, ..., n} and v is a real-valued function on the nonempty subsets of N. Elements of N are often called players while subsets of N are called coalitions. The grand coalition, denoted as N, is composed when all of the players cooperate together. The function v is called the worth function, and v(S) is interpreted as the worth of the coalition S. In other words, v(S) is the amount that individuals in S can jointly obtain if they cooperate as a group.

A game (N, v) is said to be superadditive if $v(S \cup T) \geq v(S) + v(T)$ for all disjoint coalitions $S, T \subseteq N$. The game (N, v) is monotonic if $v(S) \leq v(T)$ for all coalitions $S \subseteq T \subseteq N$. The game (N, v) is said to be 0-normalized if v(i) = 0 for all $i \in N$. The 0-normalization of a game (N, v) is the game (N, u) where

$$u(S) = v(S) - \sum_{i \in S} v(i) .$$

A game (N, v) is 0-monotonic if its 0-normalization is monotonic.

In an effort to find some standard of fairness or a predictor of bargaining solutions, many researchers of game theory have concentrated on finding the "best" way for individuals in a game to form coalitions and eventually maximize their savings. Once the savings have been made they must be allocated to the participating players. An allocation method is a function that assigns an